

A Layered Formal Methods Approach to Answering Queue-related Queries

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... imagine you are working at a cloud provider



An angry client accuses the cloud of violating a latency SLO



The cloud is only liable, iff the queue length on a port used by the client's traffic exceeded a threshold, high enough to cause a violation



The cloud is only liable, iff the queue length on a port used by the client's traffic exceeded a threshold, high enough to cause a violation

...need queue length at ms granularity



The cloud is only liable, iff the queue length on a port used by the client's traffic exceeded a threshold, high enough to cause a violation



...need queue length at ms granularity
...have packet counts at min granularity



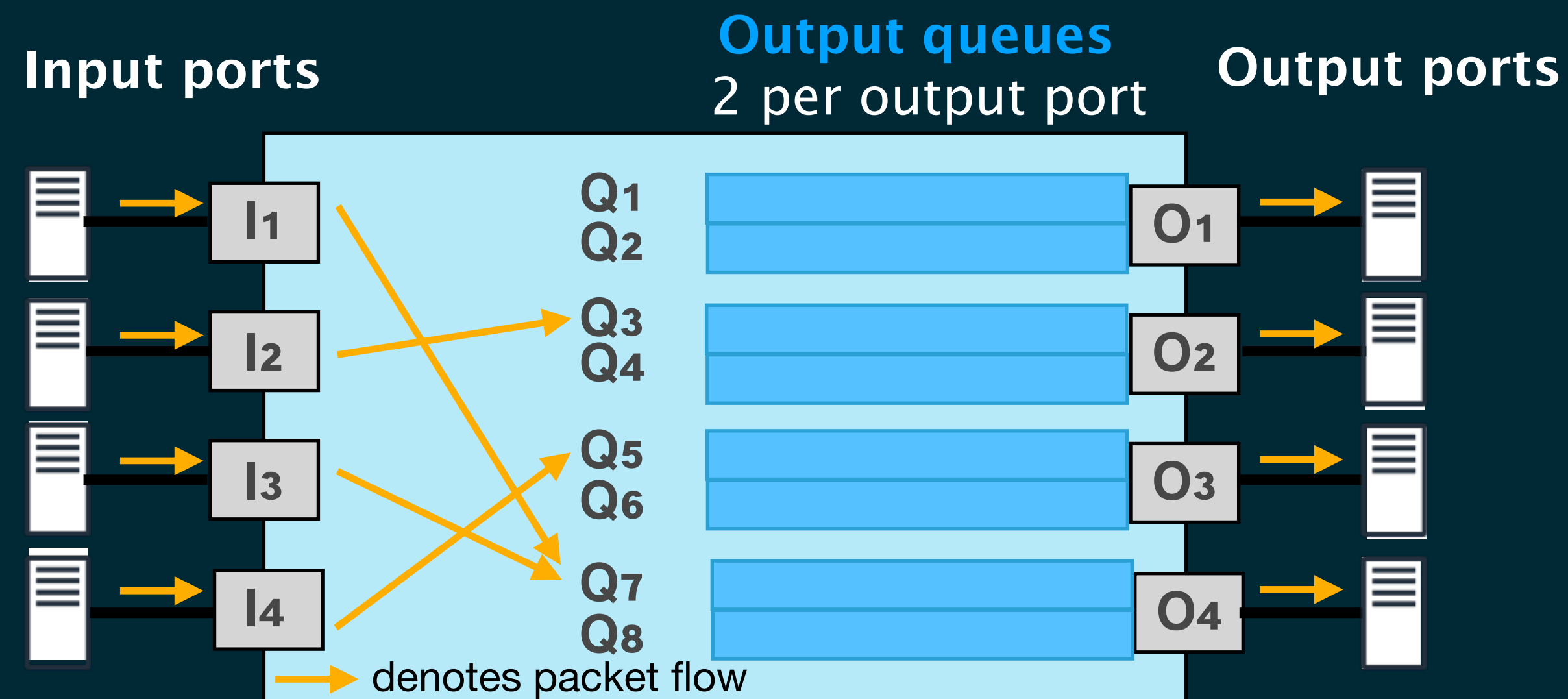
Can one still prove that the latency SLO was not violated,
using only packet counts?

Can one answer queue-related queries,
using only packet counts?

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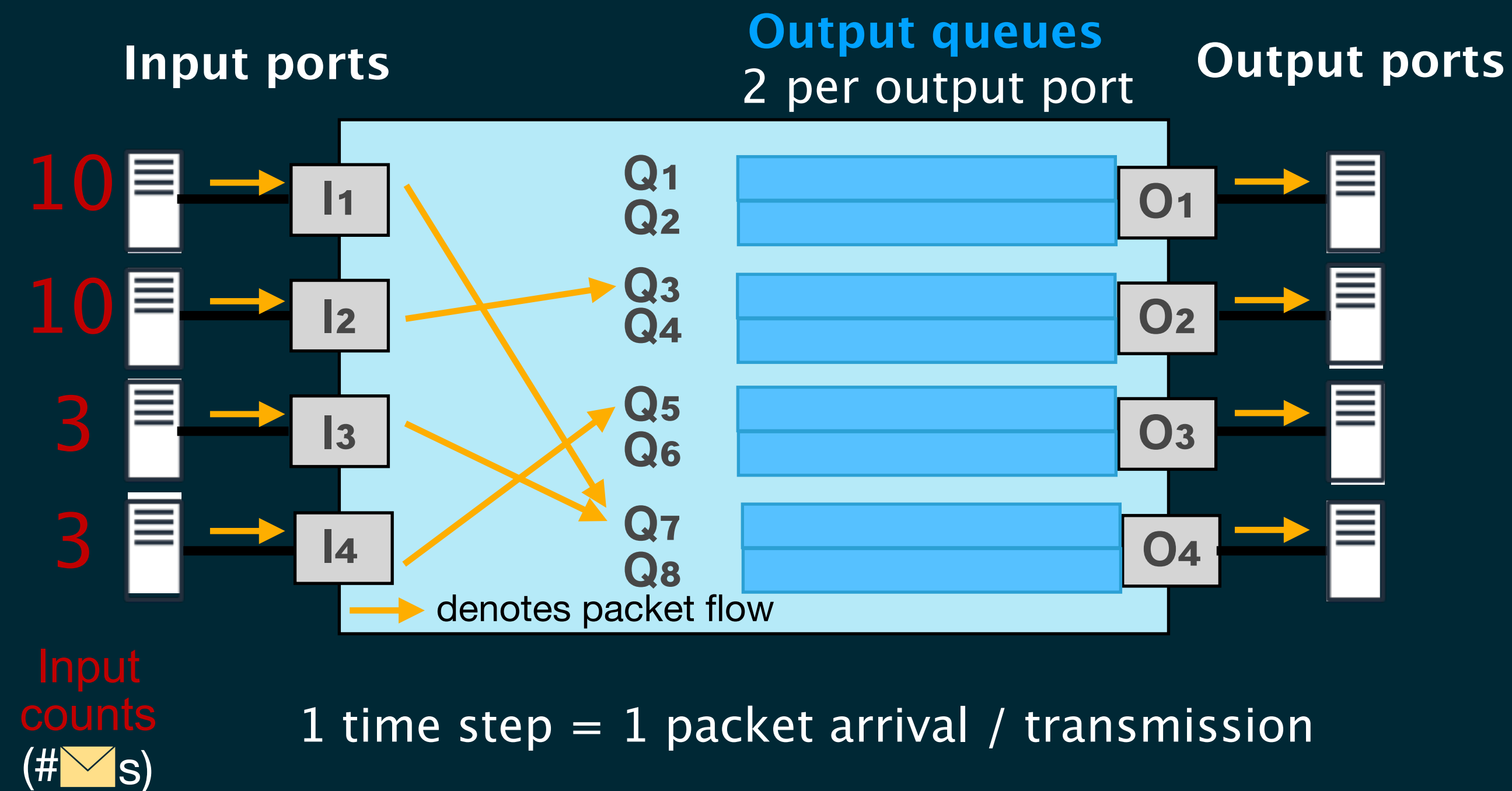
Hint: There is a connection between packet counts and queue lengths

The network operator has access to per-port input, output and dropped packet counts but needs to reason about queue lengths



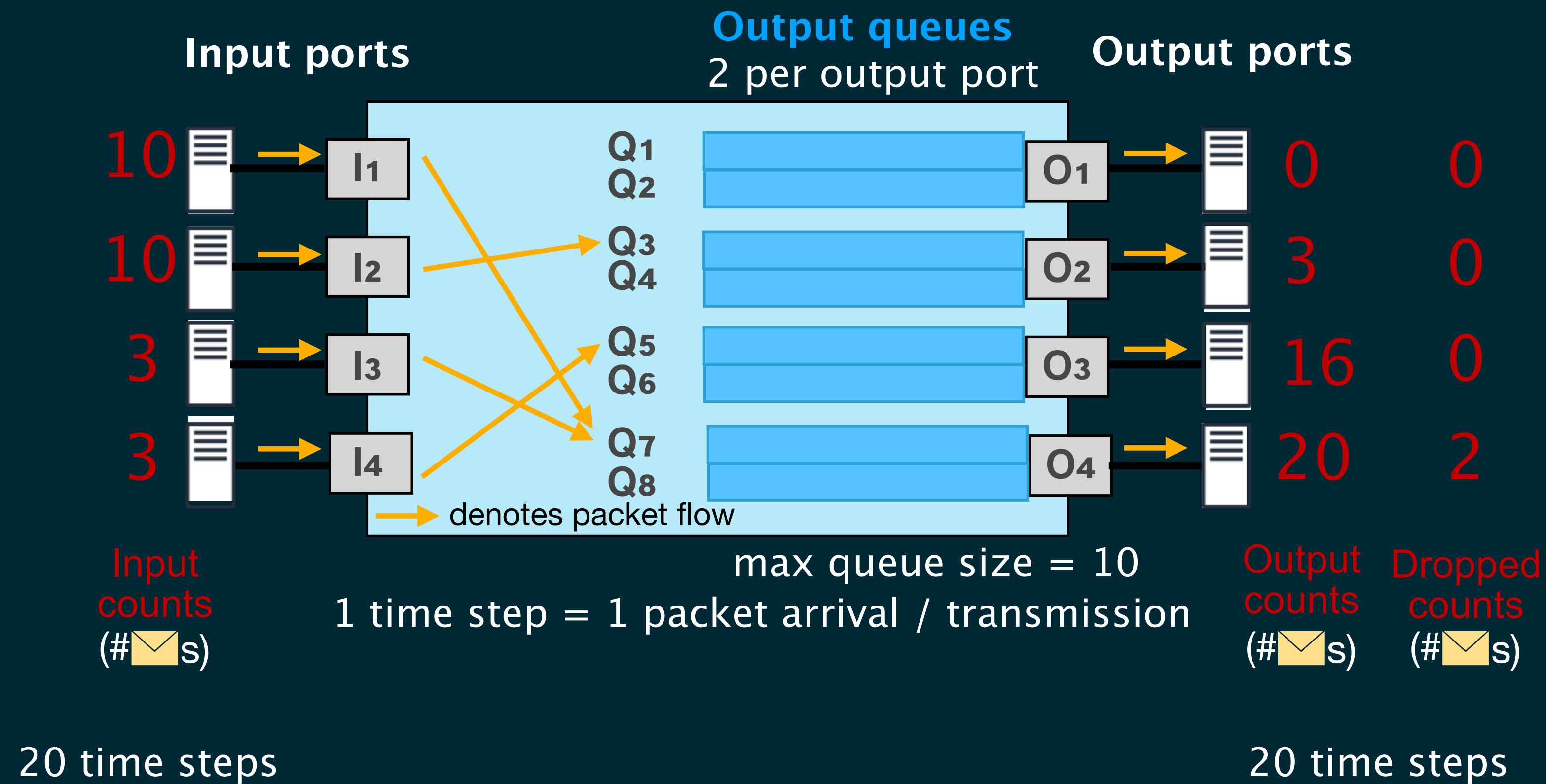
1 time step = 1 packet arrival / transmission

The network operator has access to per-port input, output and dropped packet counts but needs to reason about queue lengths

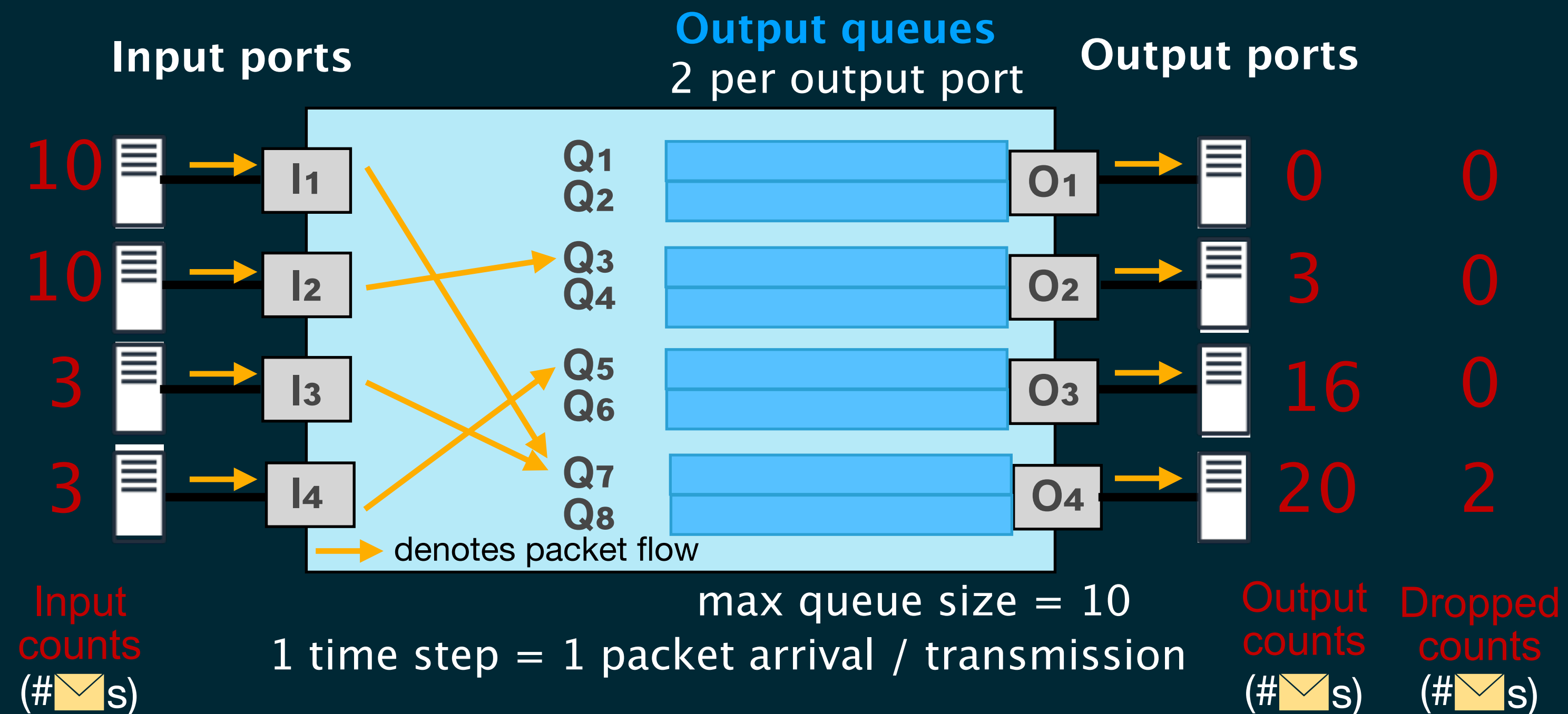


20 time steps

The network operator has access to per-port input, output and dropped packet counts but needs to reason about queue lengths

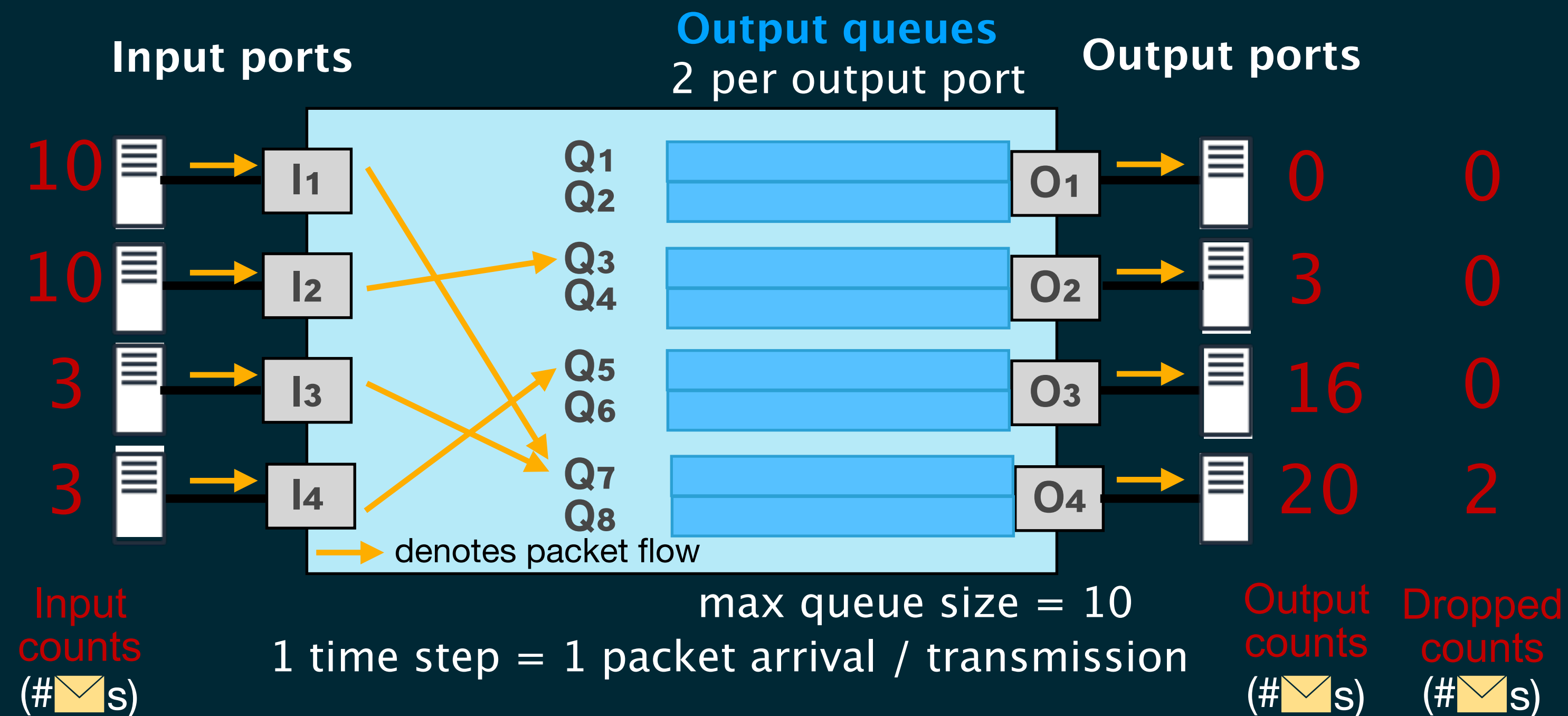


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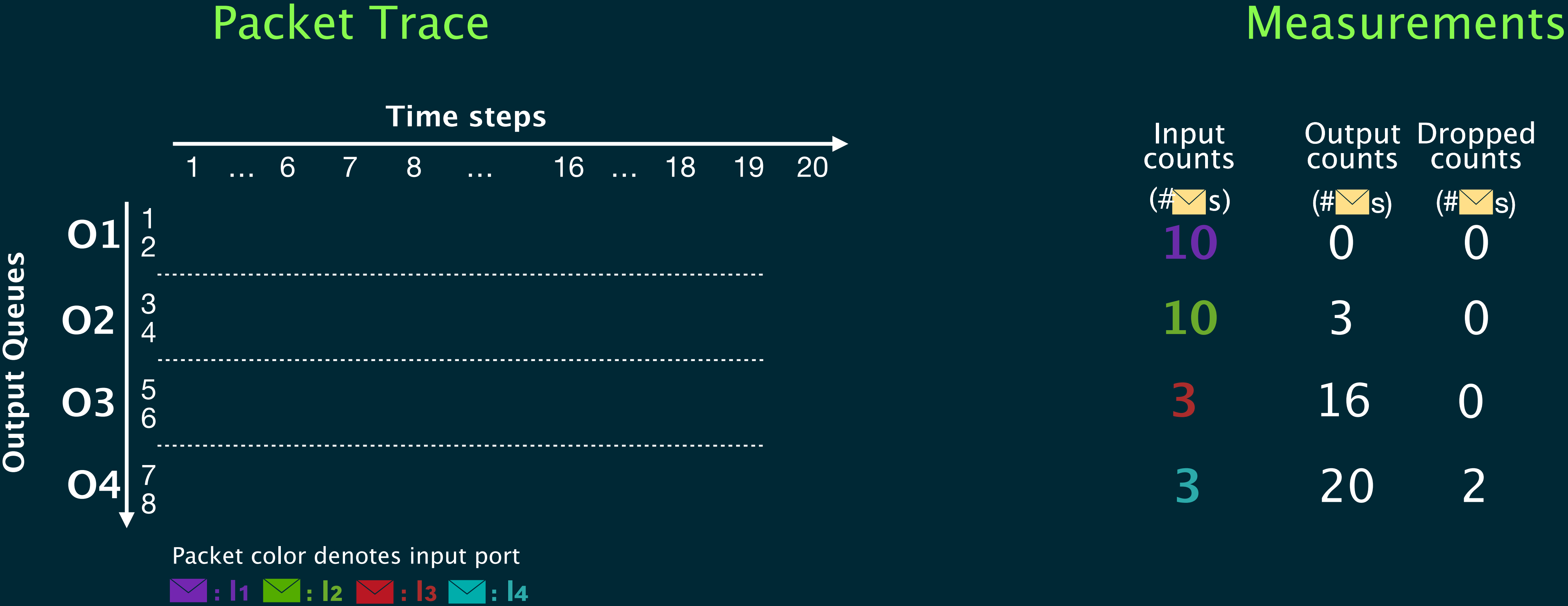
Can queue size of port O_2 be ≥ 7 ?

The network operator needs to find a sequence of packet arrivals, that satisfy the packet counts and lead to such a queue length



Can queue size of port O_2 be ≥ 7 ?

The network operator needs to find a **Packet Trace**,
that satisfy the packet counts and lead to a queue length of 7 pkts

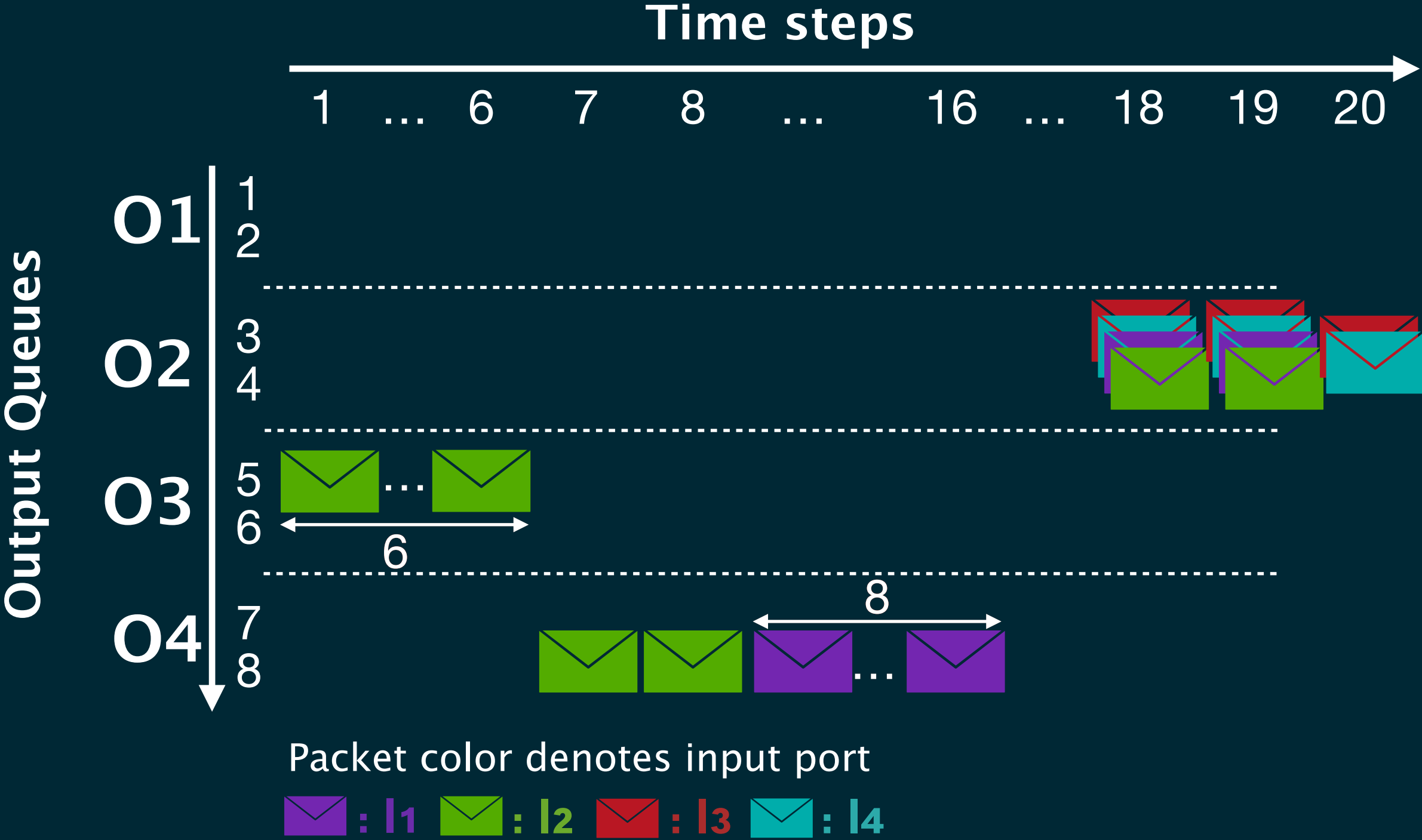


The network operator needs to find a **Packet Trace**,
that satisfy the packet counts and lead to a queue length of 7 pkts



The operator can start with input counts and generate a packet trace that will satisfy those and the query ($O_2 > 7$ pkts)

Packet Trace



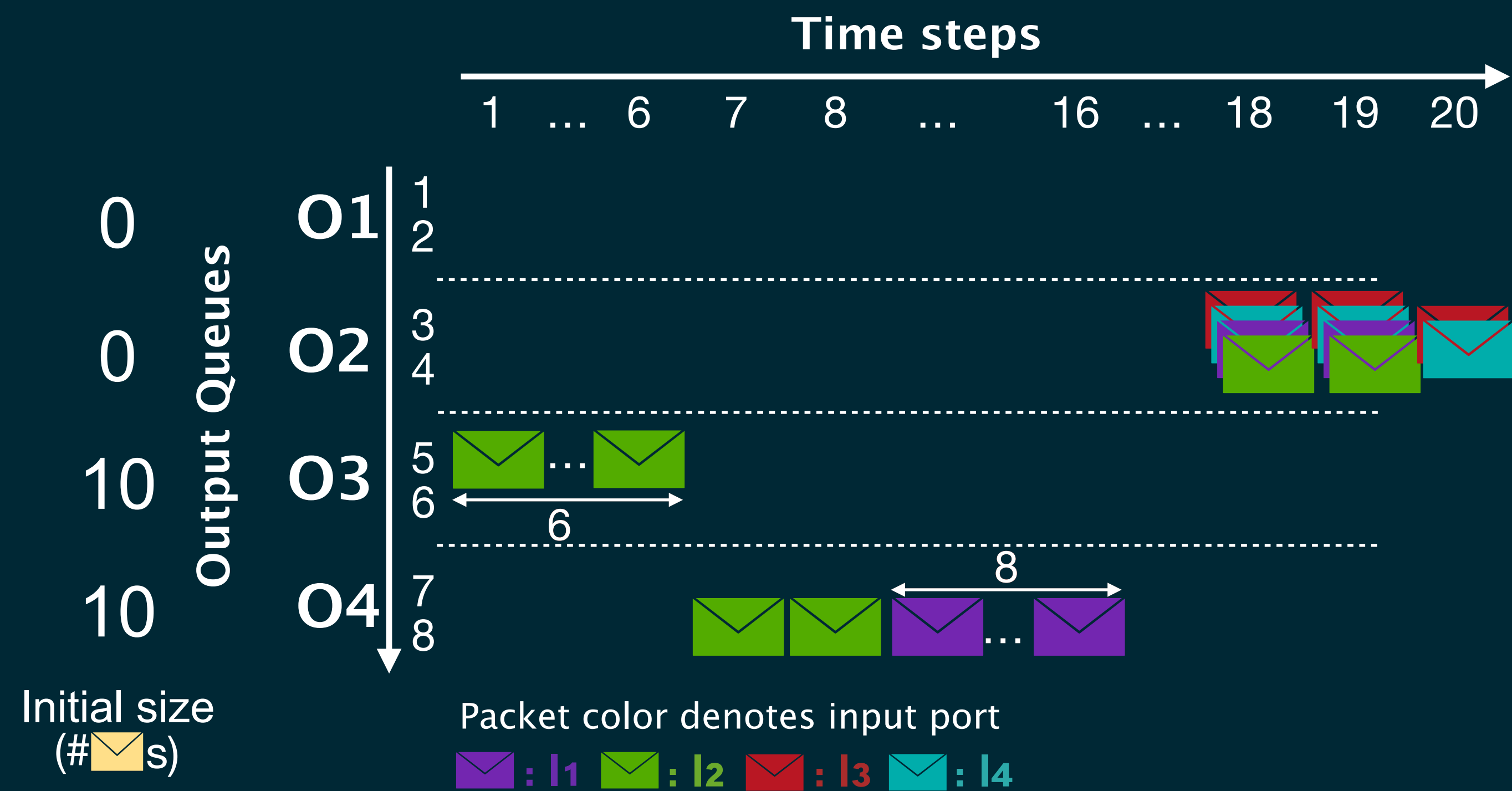
Measurements

Input counts (#s)	Output counts (#s)	Dropped counts (#s)
10	0	
10	3	
3	16	
3	20	

✓

The packet trace is consistent with input, output counts and the query

Packet Trace

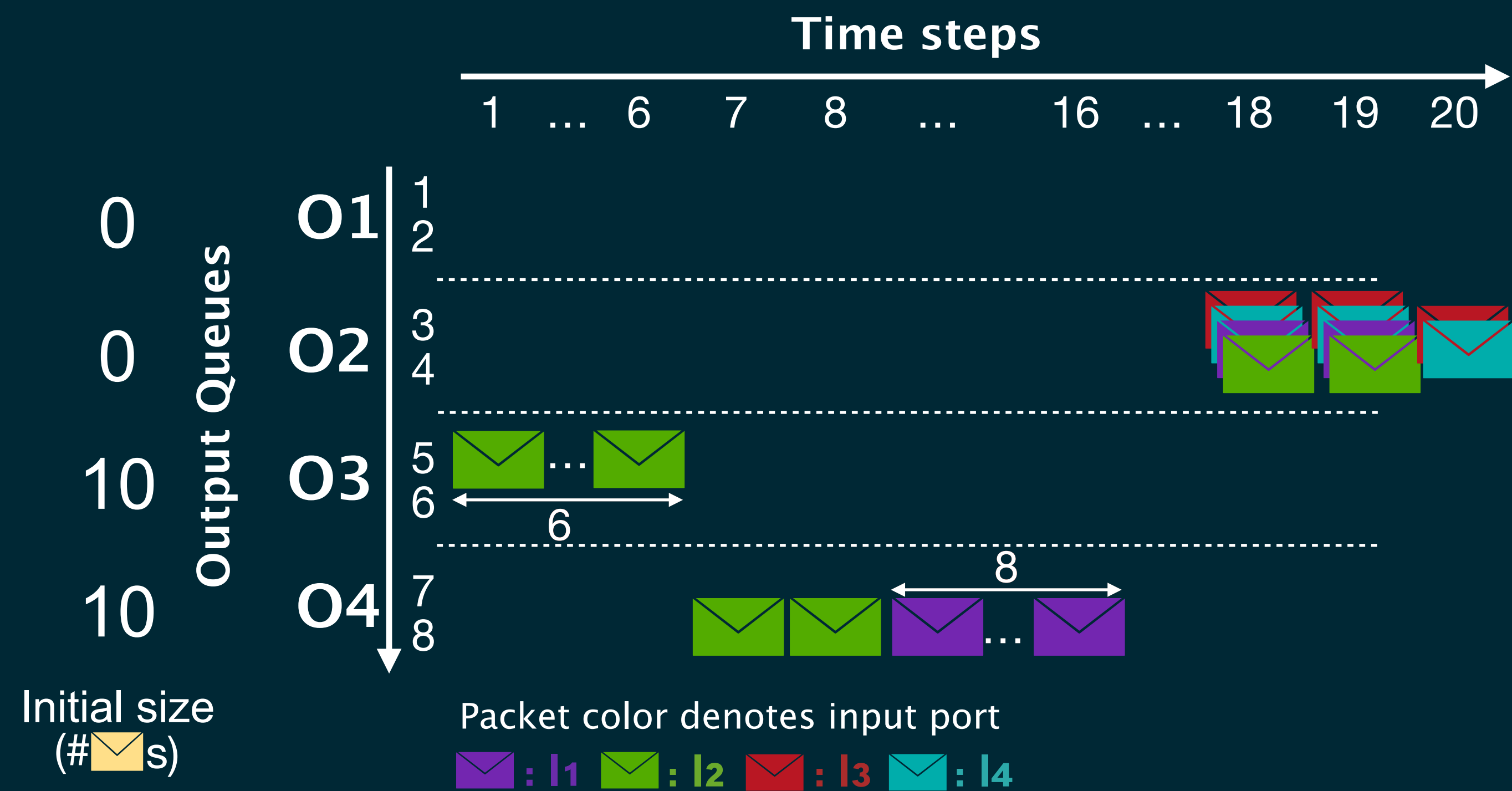


Measurements

Input counts (#📧s)	Output counts (#📧s)	Dropped counts (#📧s)
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✓	✓	

The packet trace is consistent with input, output counts and the query but not with drop counts

Packet Trace



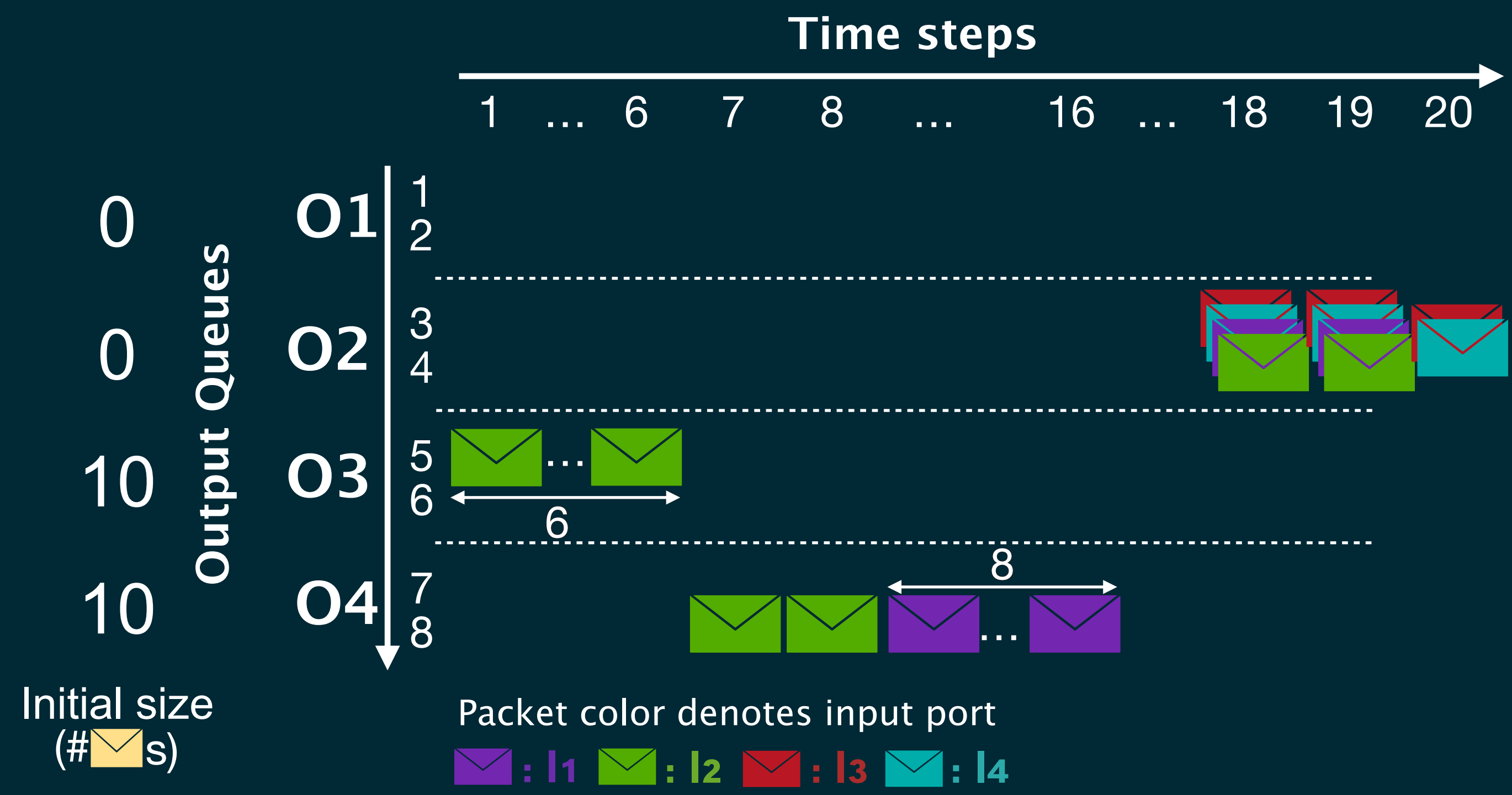
Measurements

Input counts (#📧s)	Output counts (#📧s)	Dropped counts (#📧s)
10	0	0
10	3	0
3	16	0
3	20	2
✓	✓	✗

The packet trace is consistent with input, output counts and the query but not with drop counts, **let's start over**

Packet Trace

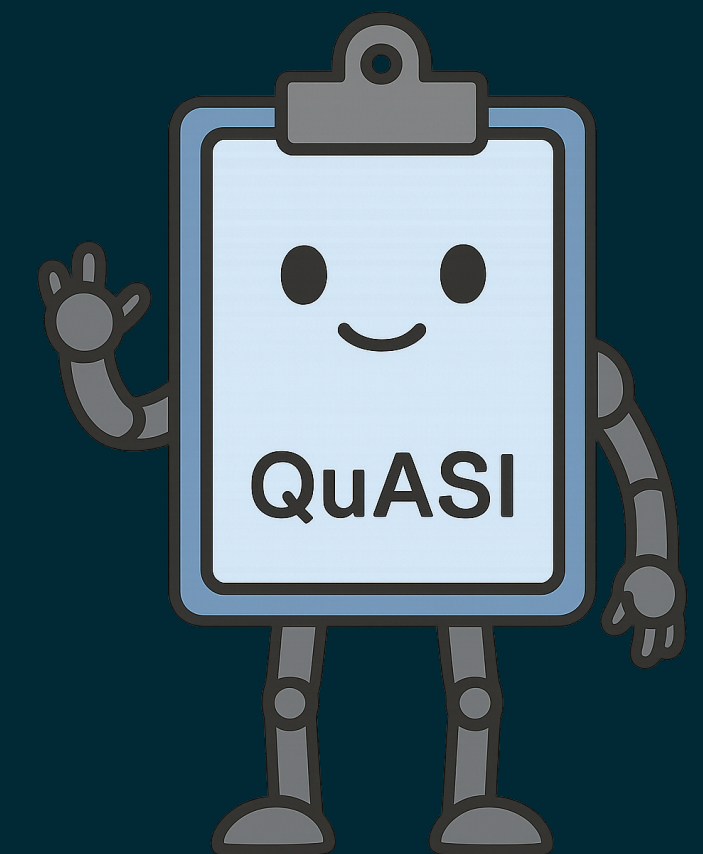
Measurements



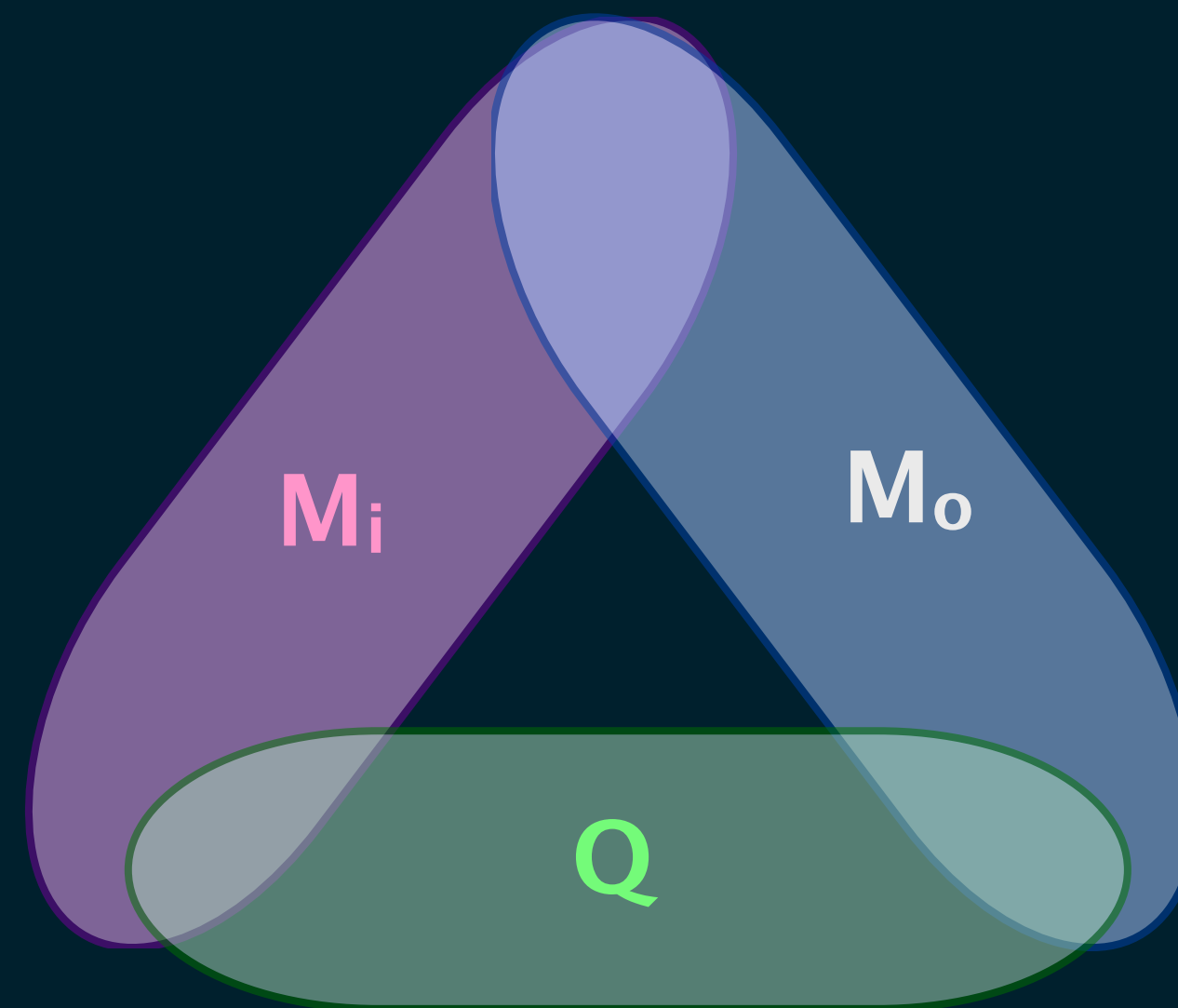
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There are $\approx 10^{16}$ distinct packet traces that one can generate from the input counts...
proving impossibility requires checking all of them

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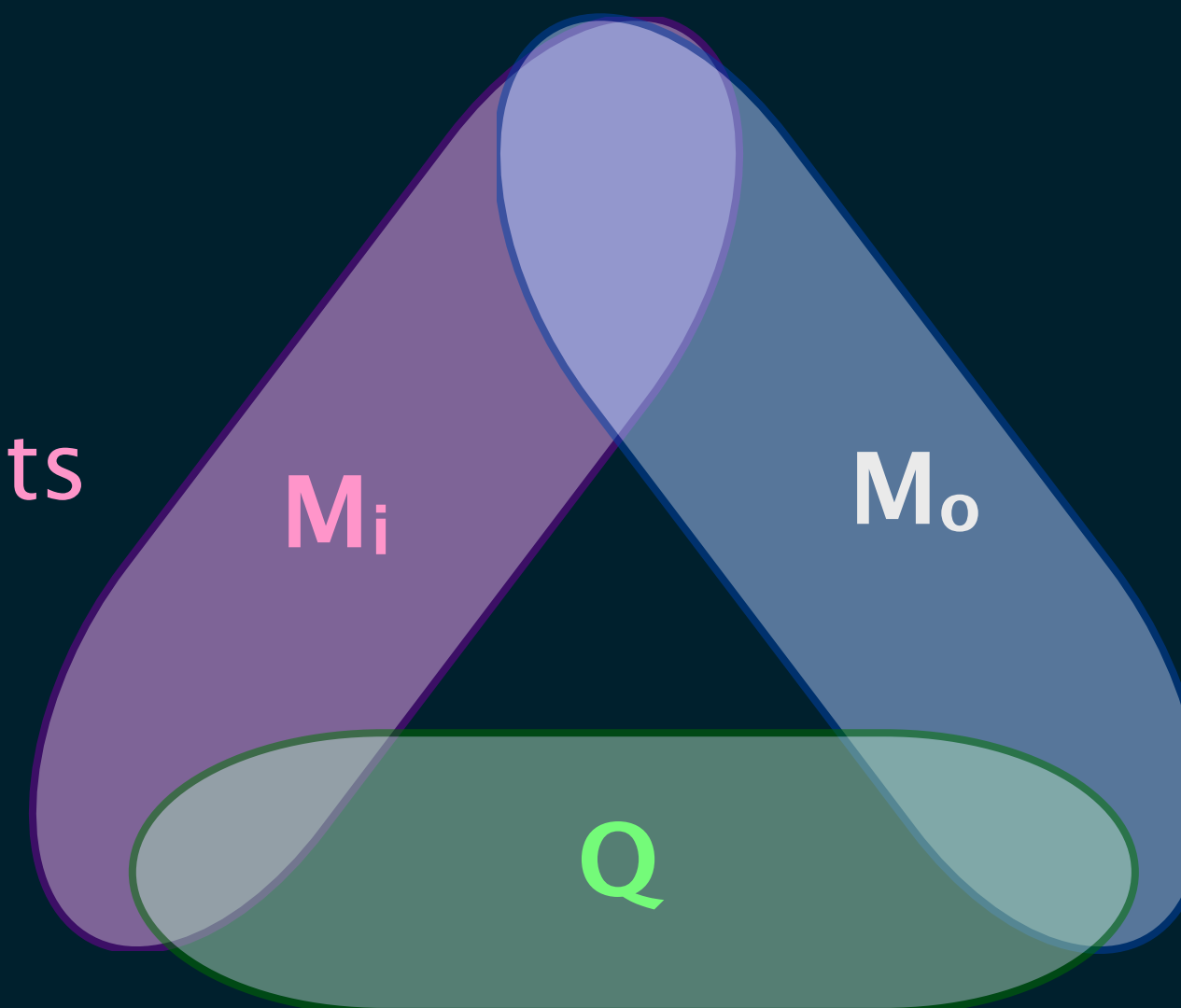


QuASI answers queue-related queries
using per-port incoming, outgoing and dropped packet counts



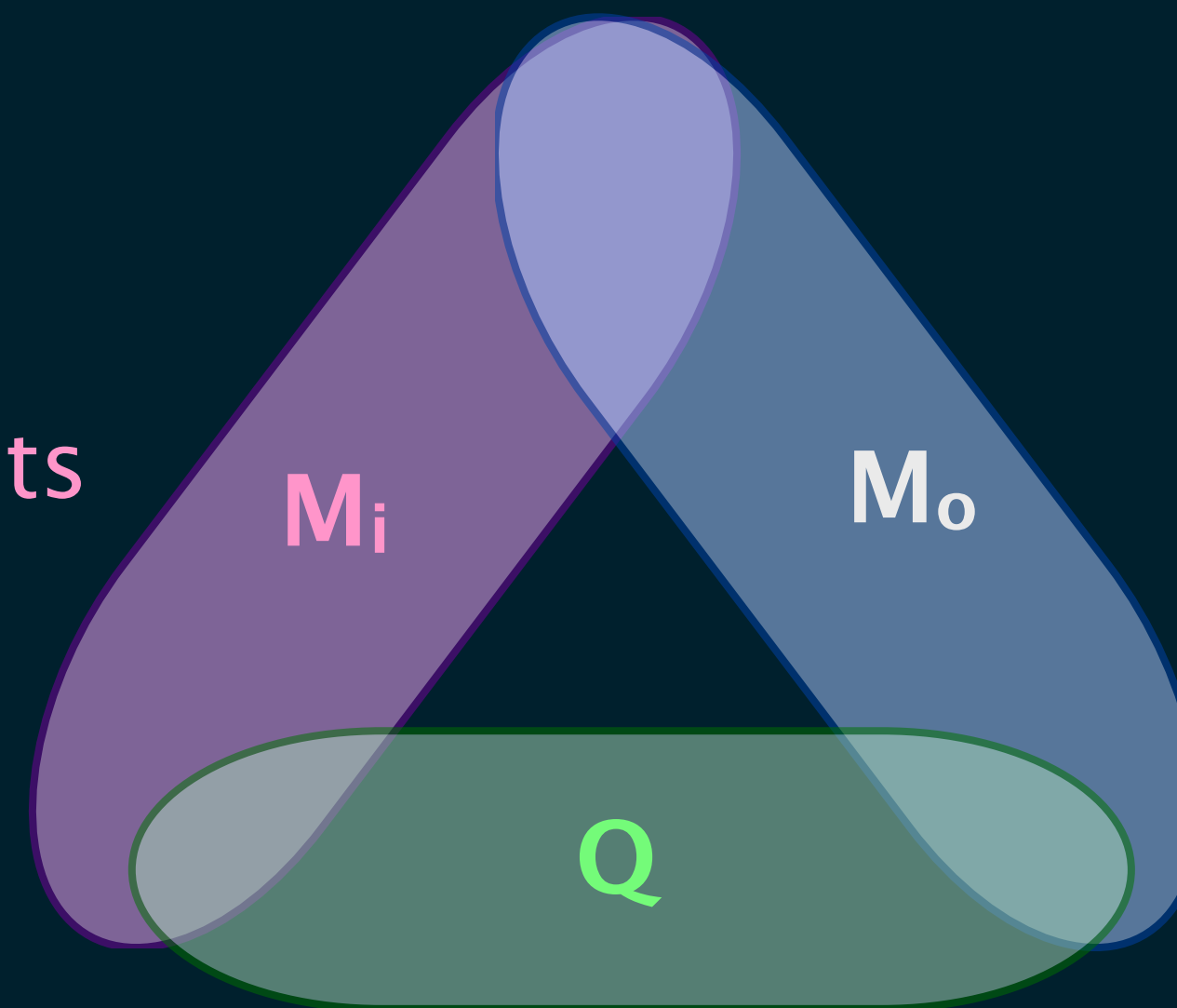
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Set of packet traces
consistent with input counts



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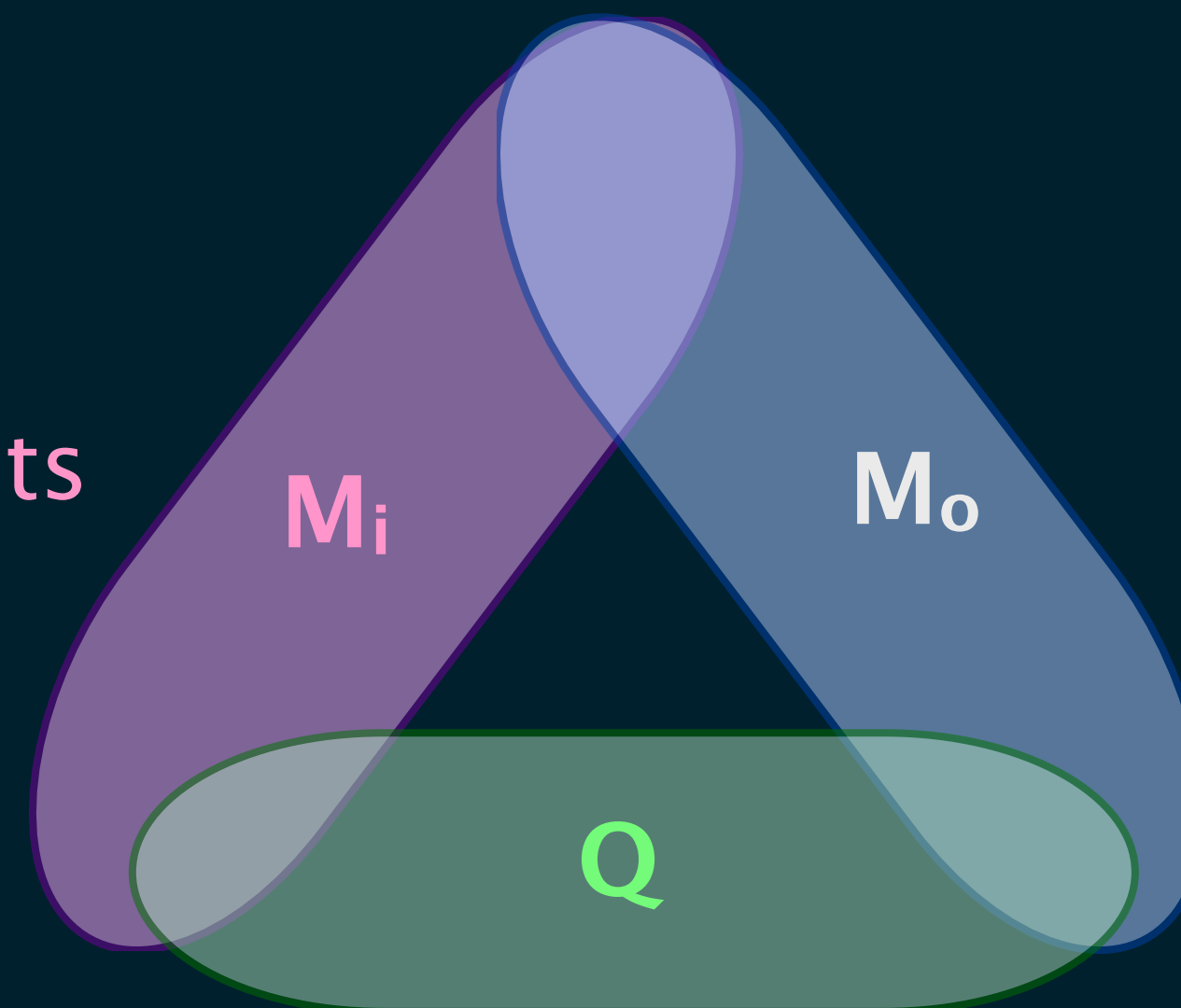
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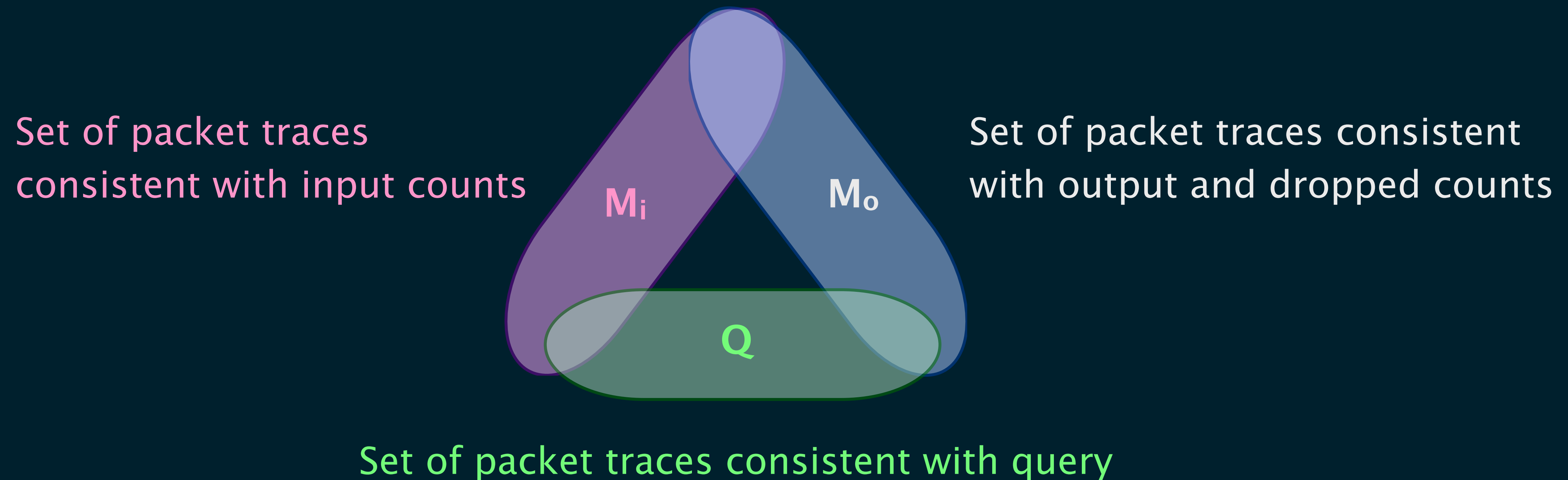
Set of packet traces
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Set of packet traces consistent
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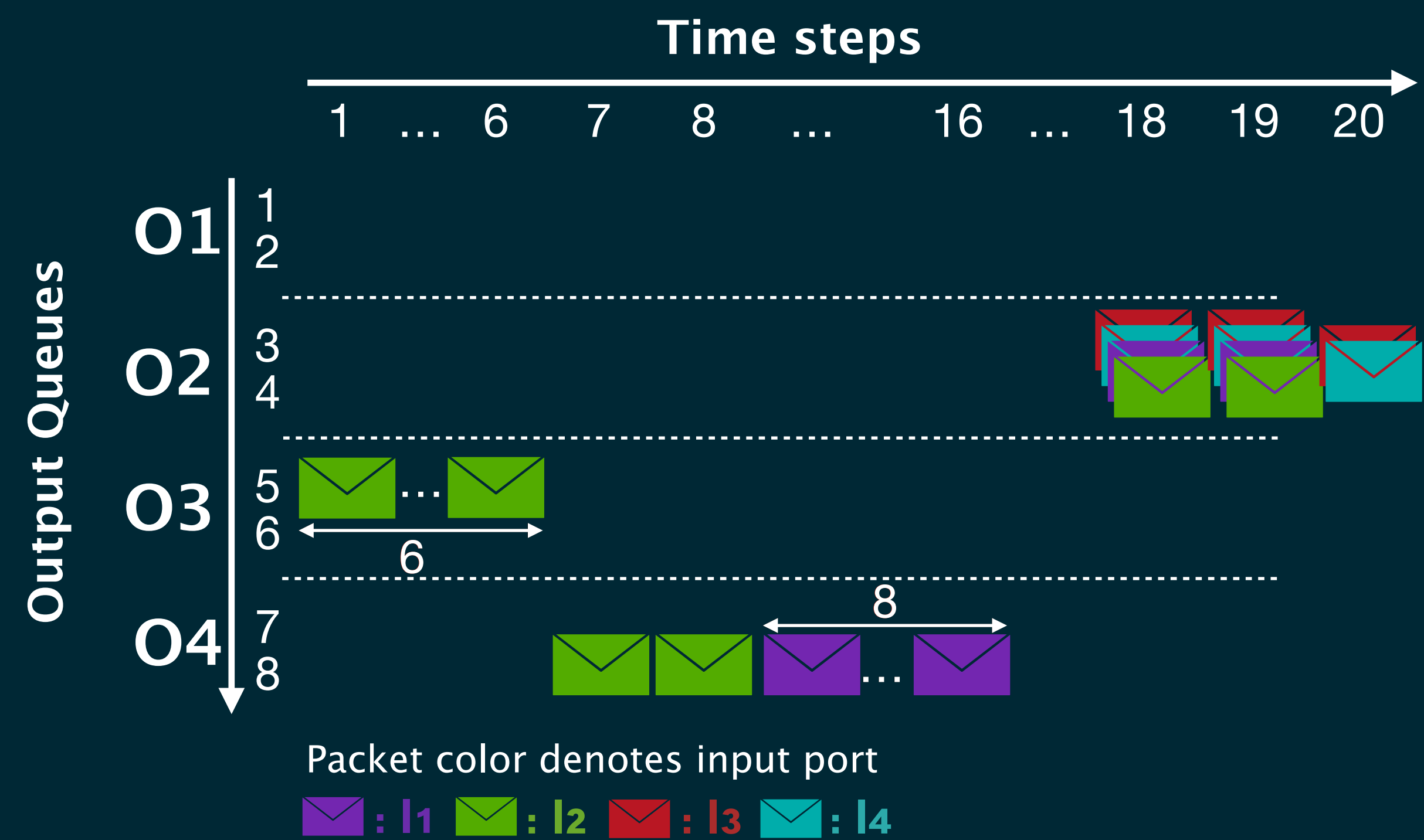
Set of packet traces consistent with query

QuASI answers queue-related queries
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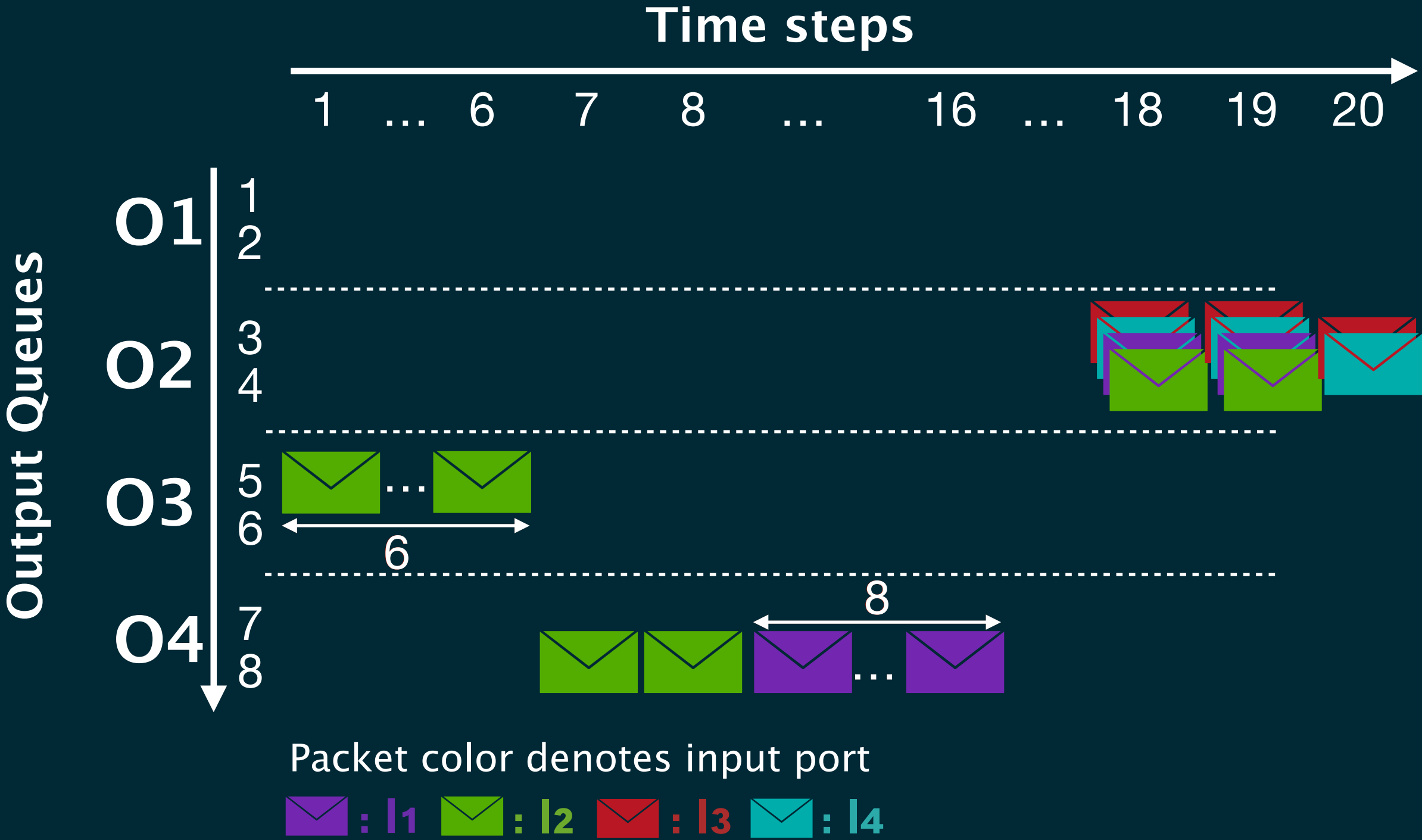
QuASI checks if there is a packet trace in the intersection of those sets

Instead of looking at each packet trace,
QuaSi relies on an enqueue abstraction



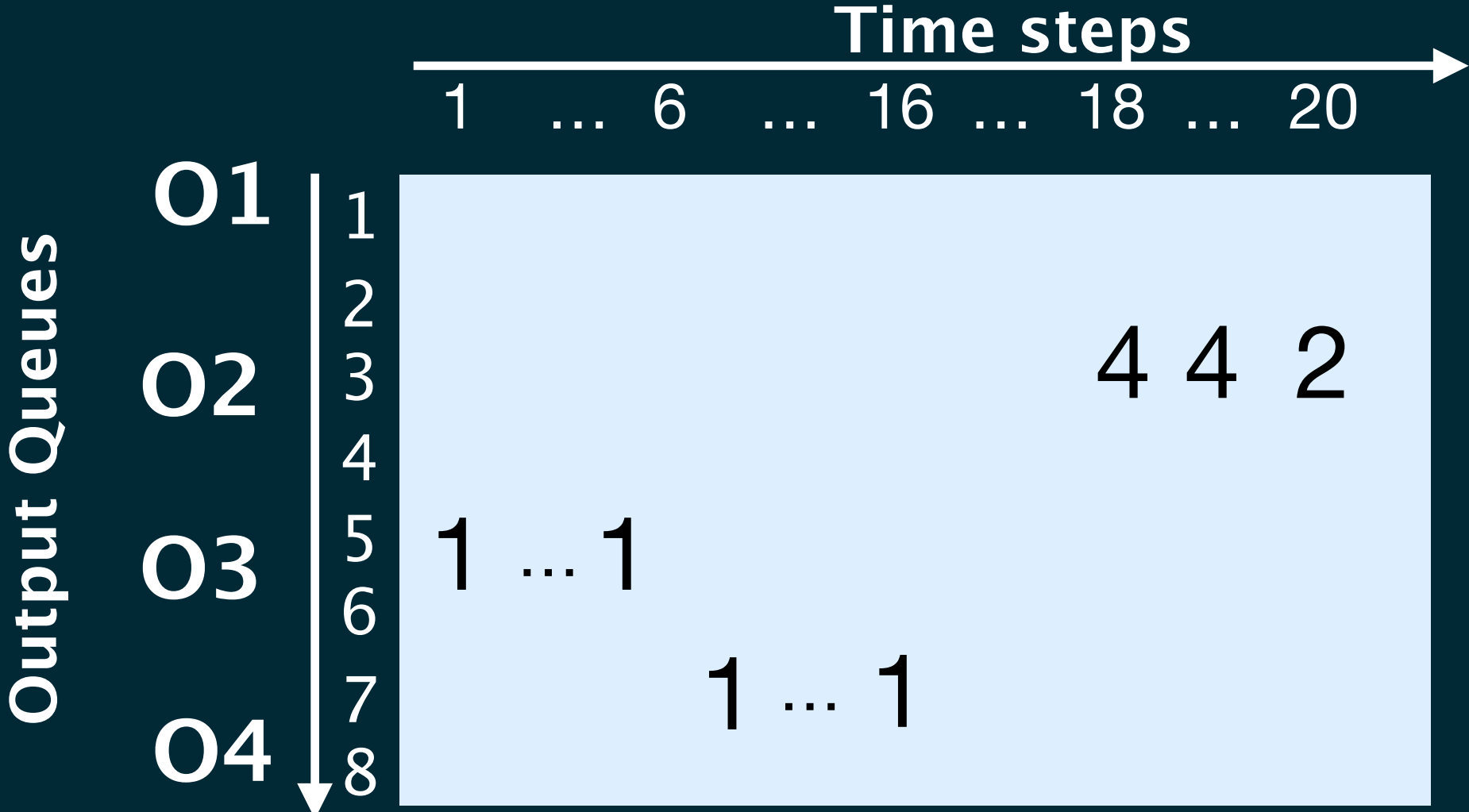
Packet trace

QuaSi reasons about **abstract packet traces**:
number of packets that were enqueued per queue and time step



Packet trace

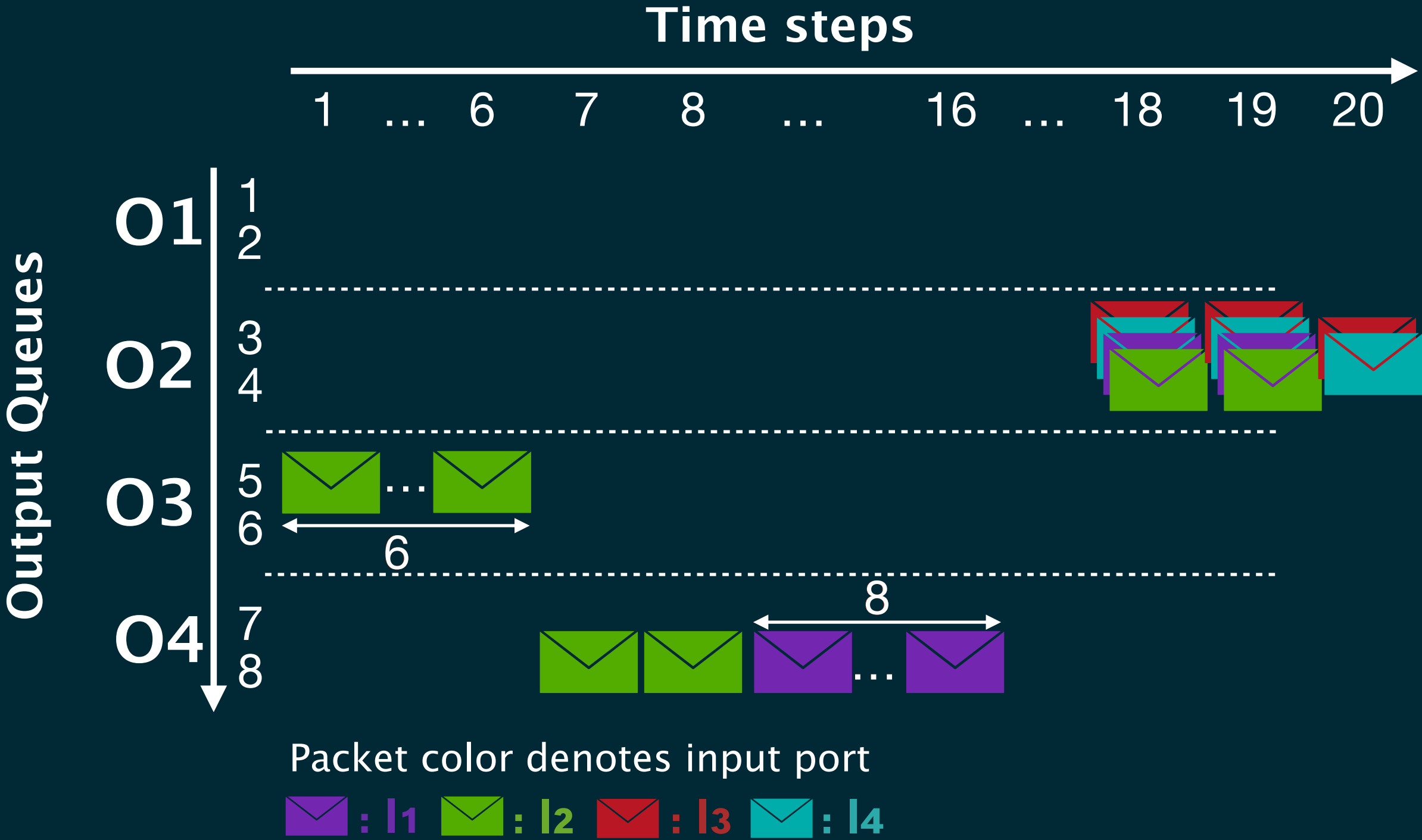
Enqueue-rate
Abstraction



Blank entries are 0

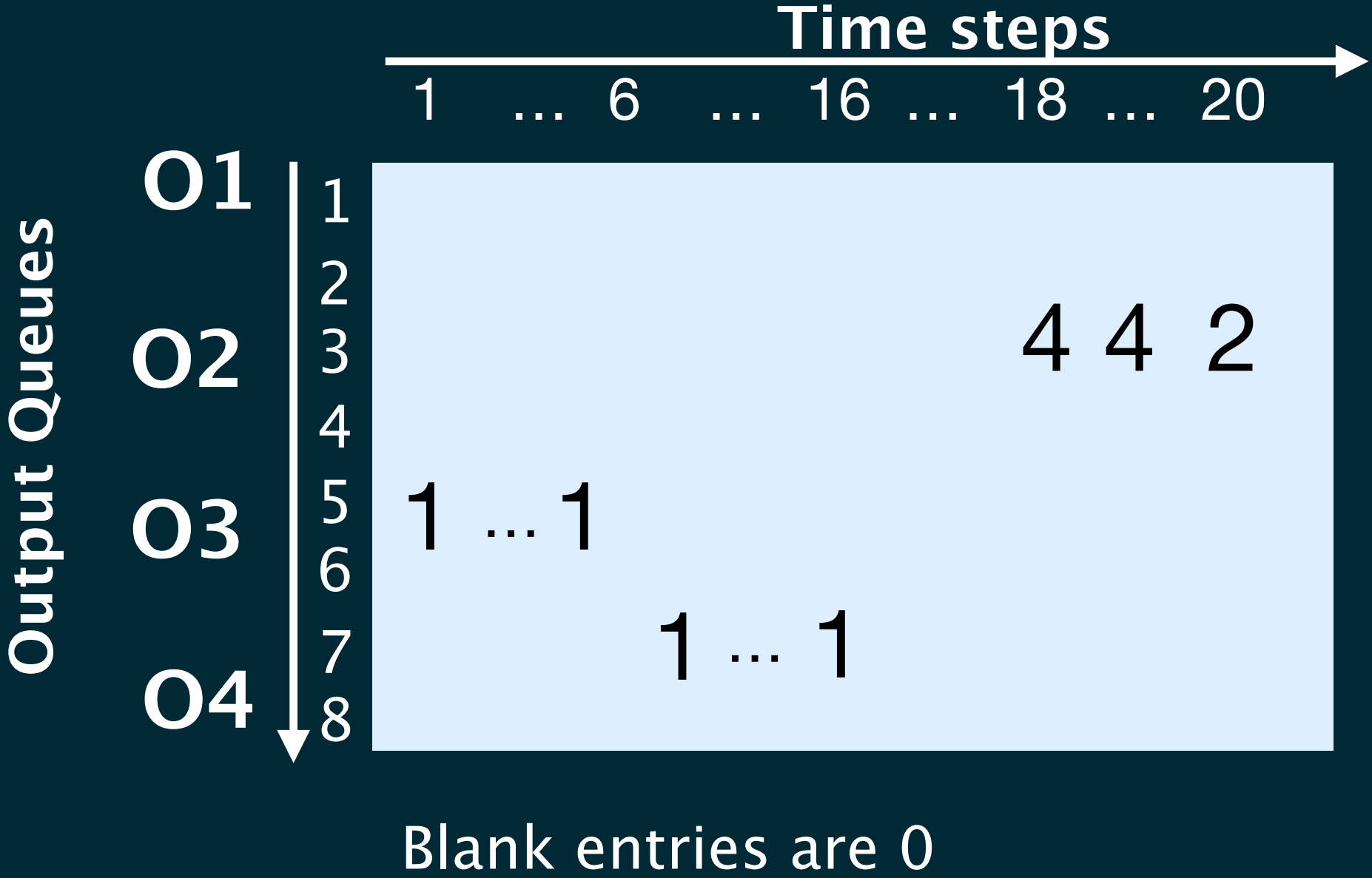
Abstract packet trace

The enqueue abstraction is **lossless** for the queries we support



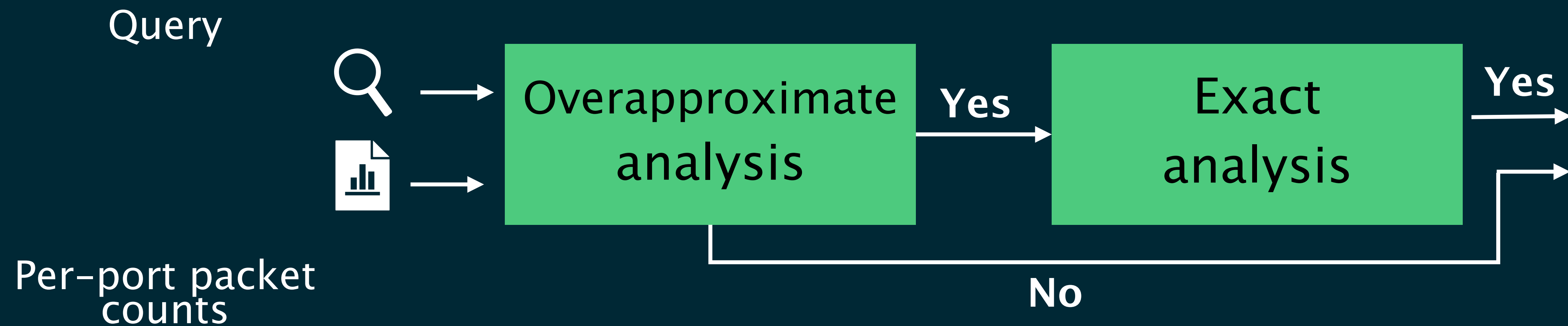
Packet trace

Enqueue-rate
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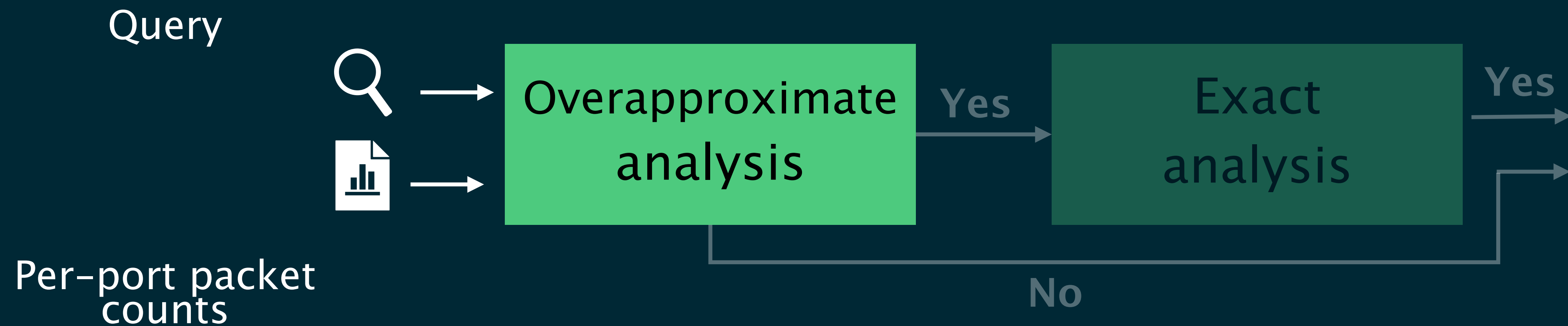


Abstract packet trace

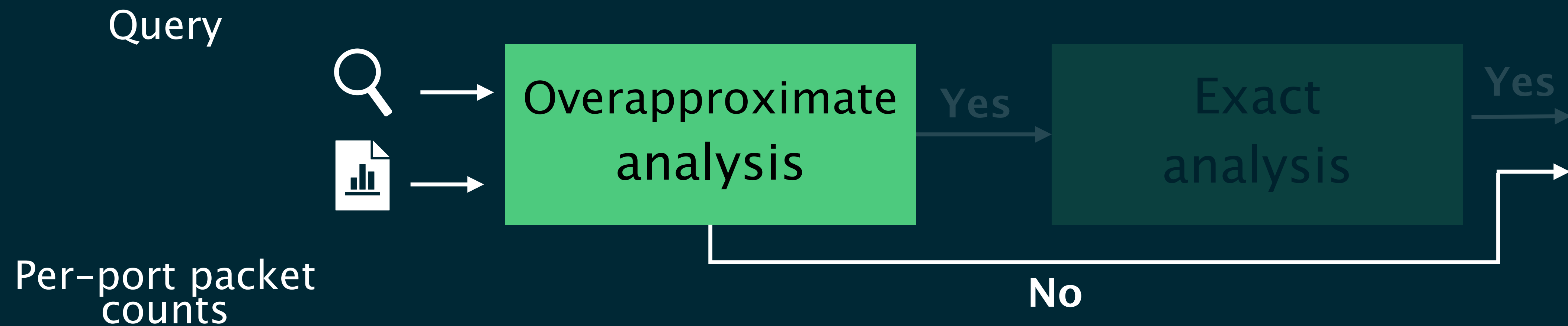
QuASI is inspired by abstraction refinement



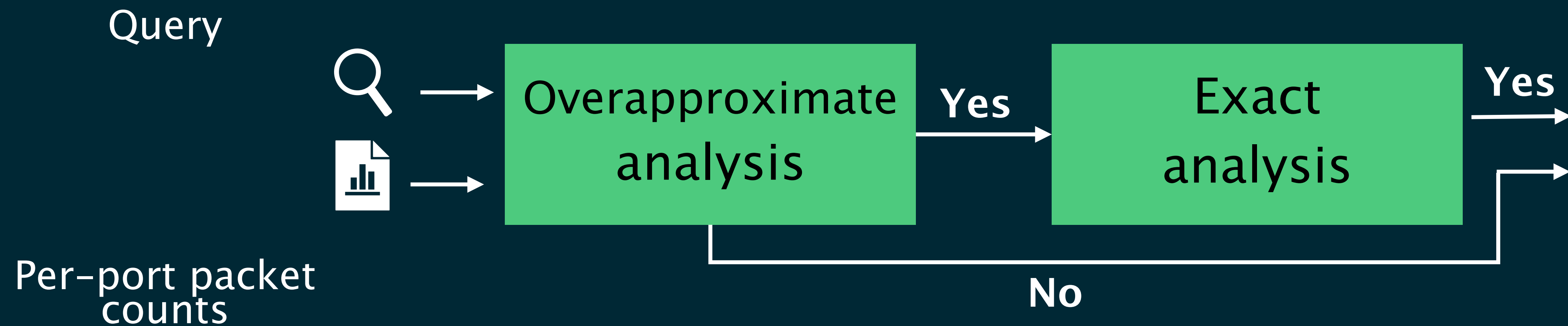
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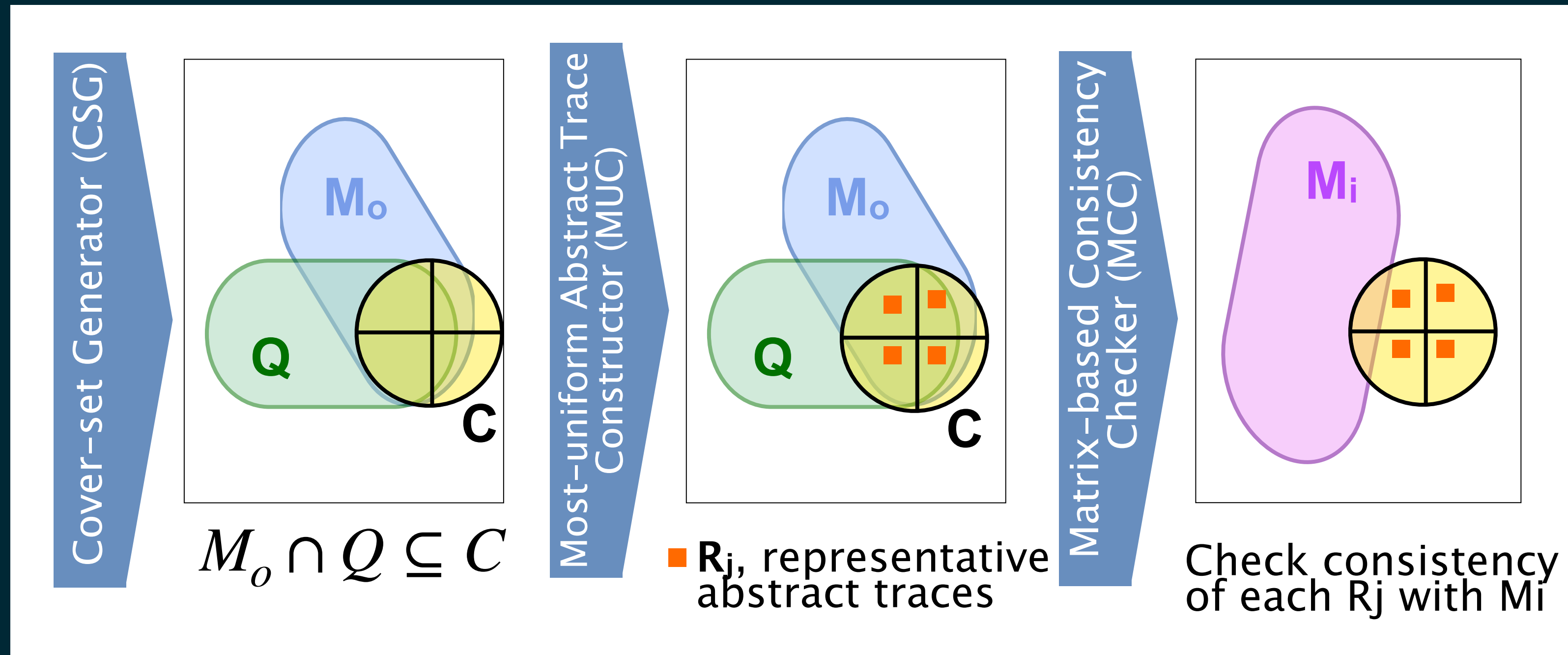


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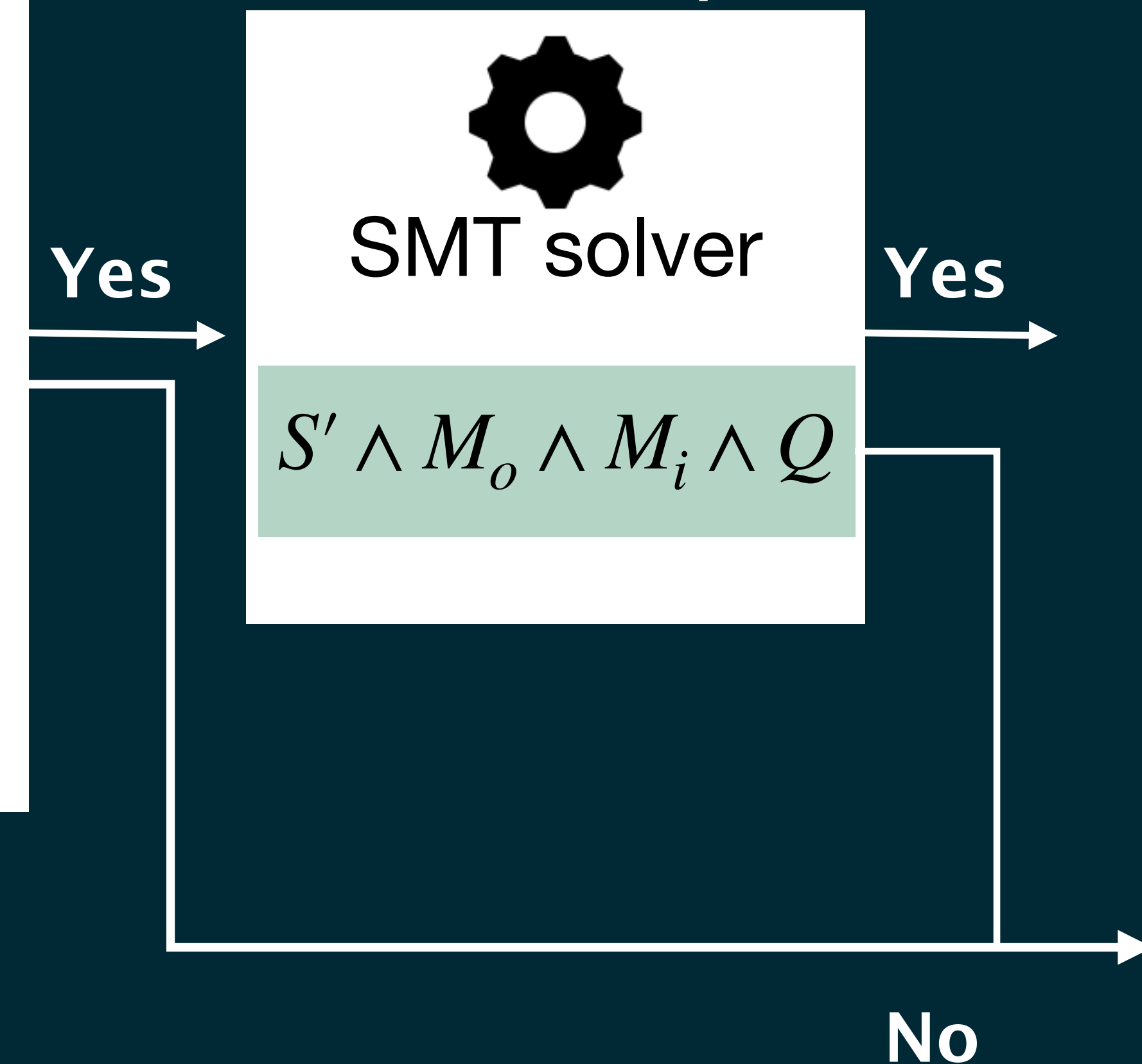


QuASI is inspired by abstraction refinement

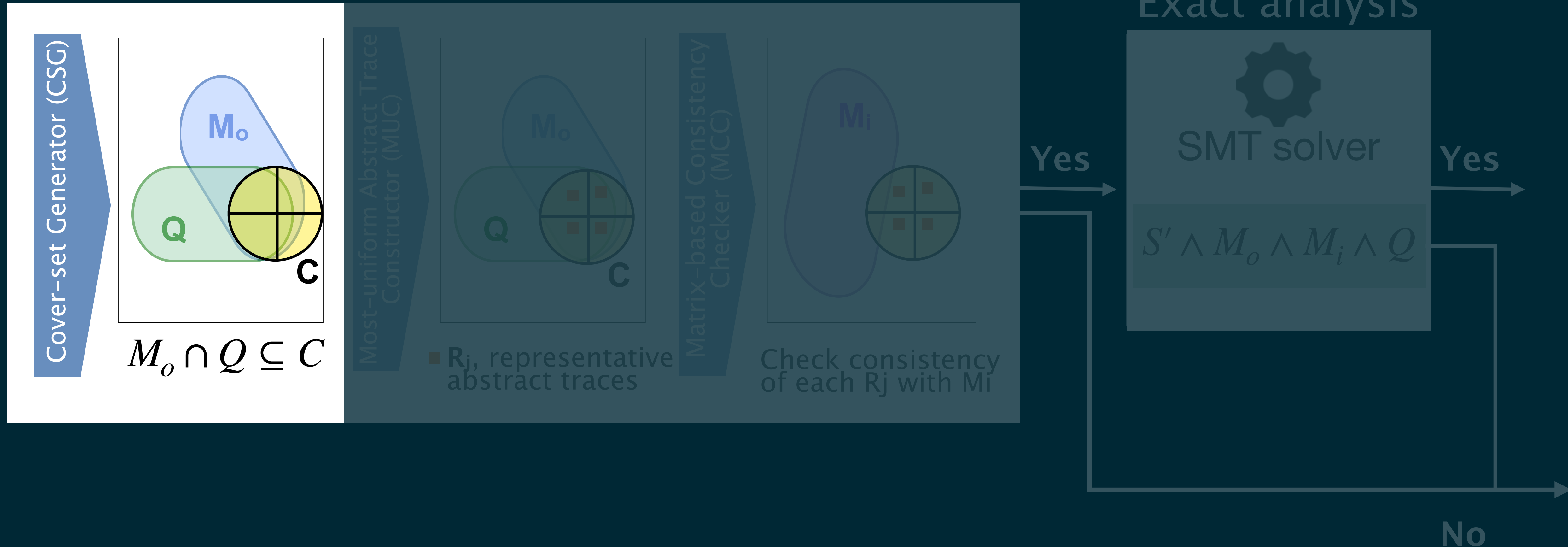
Over-approximate analysis



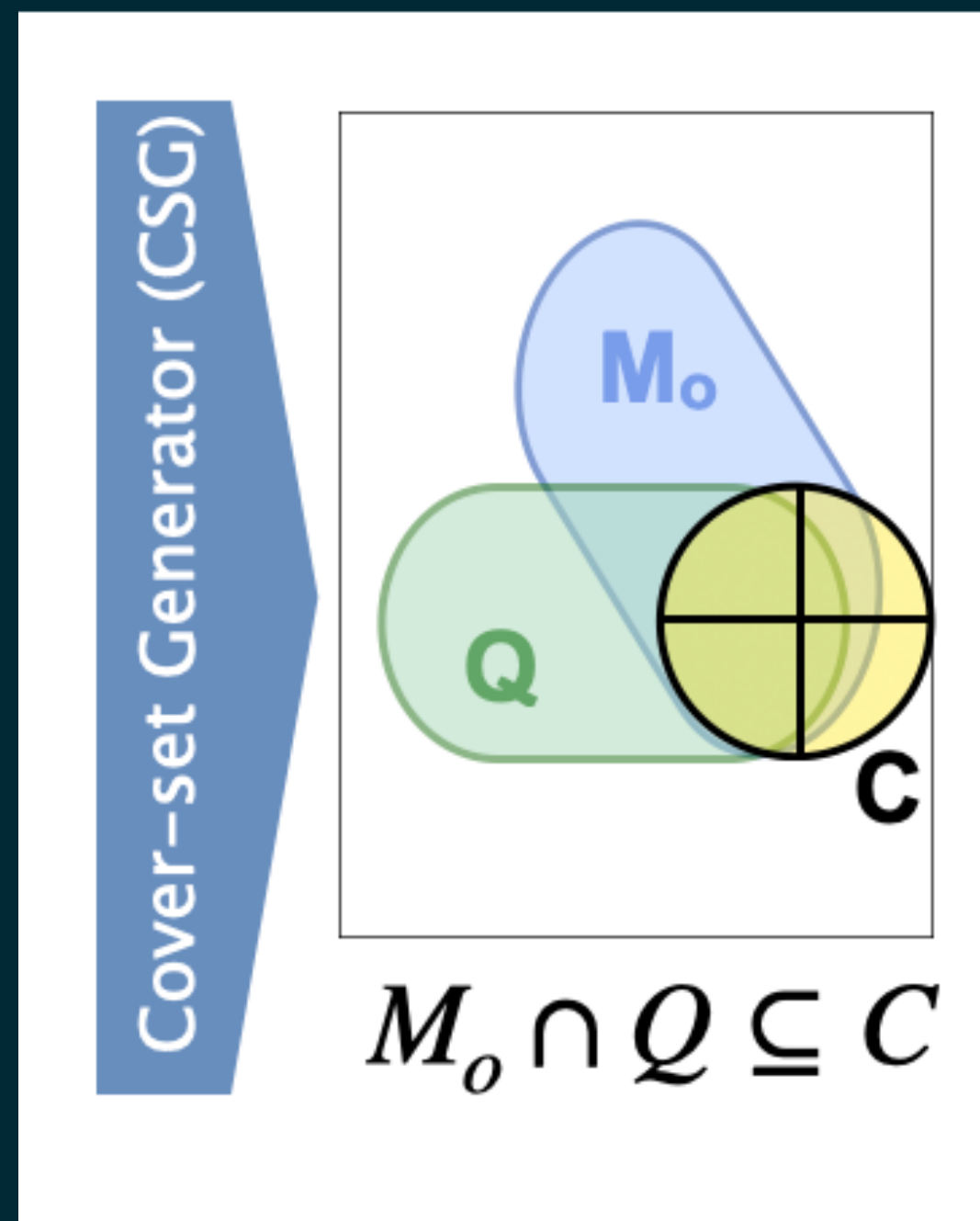
Exact analysis



Over-approximate QuASI step 1: The cover-set generator finds necessary conditions for consistency with output counts M_o and the Query Q



Over-approximate QuASI step 1: Cover-set is a disjunction of components, each being a conjunction of expressions over enqueue rate



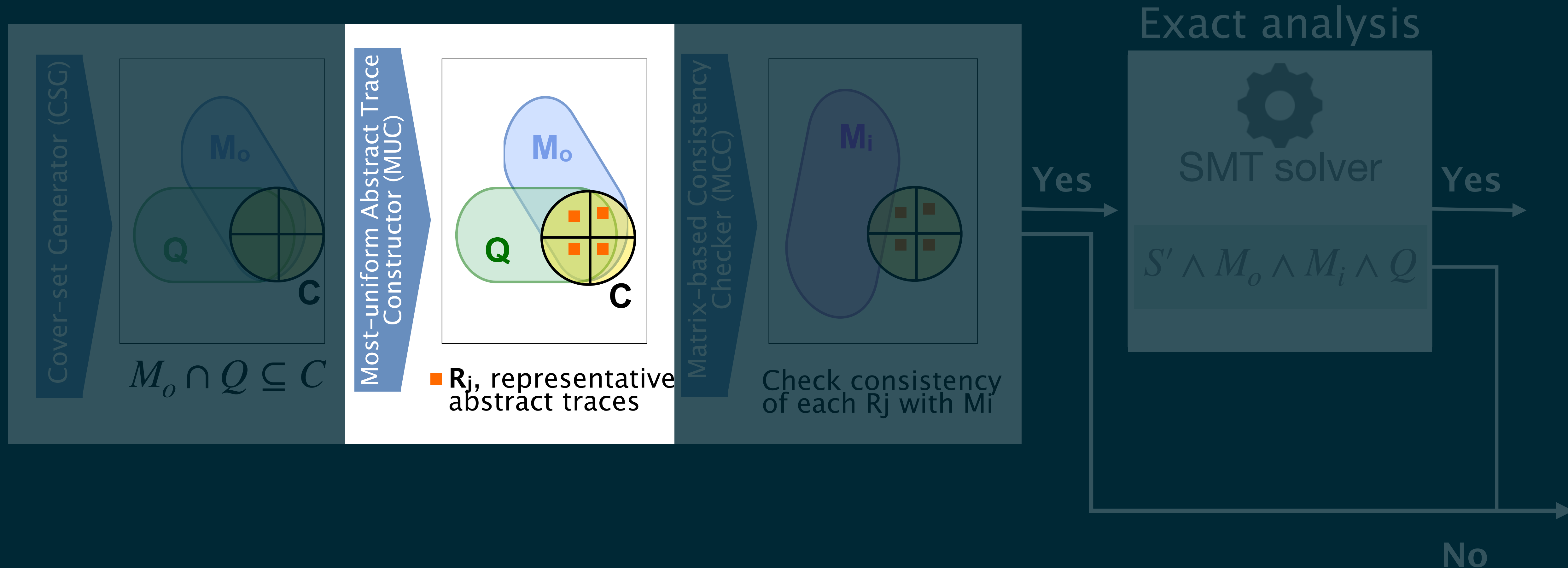
$$\sum_{q \in O_2} cenq(q, T) \geq 10 \wedge \sum_{q \in O_3} cenq(q, T) \geq 6 \wedge \sum_{q \in O_4} cenq(q, T) \geq 12$$

F_C^k

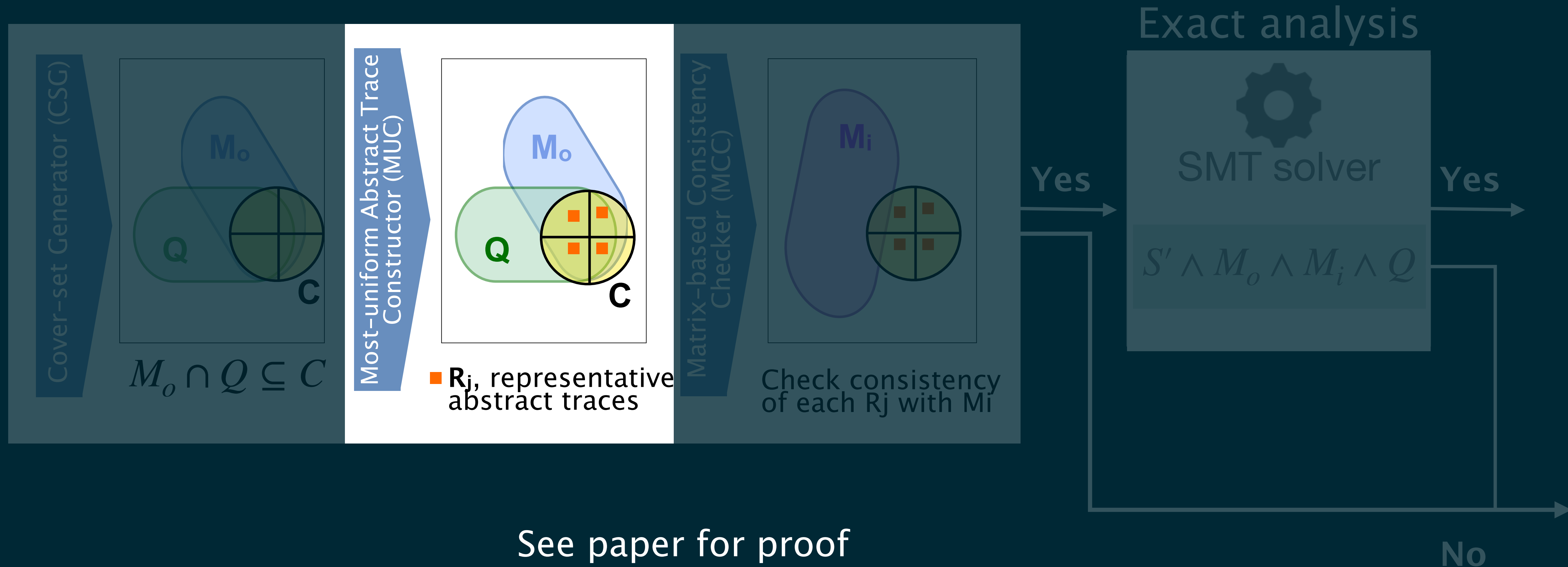
F_C^1

Each component F_C^i is conjunction of expressions over enqueue rate

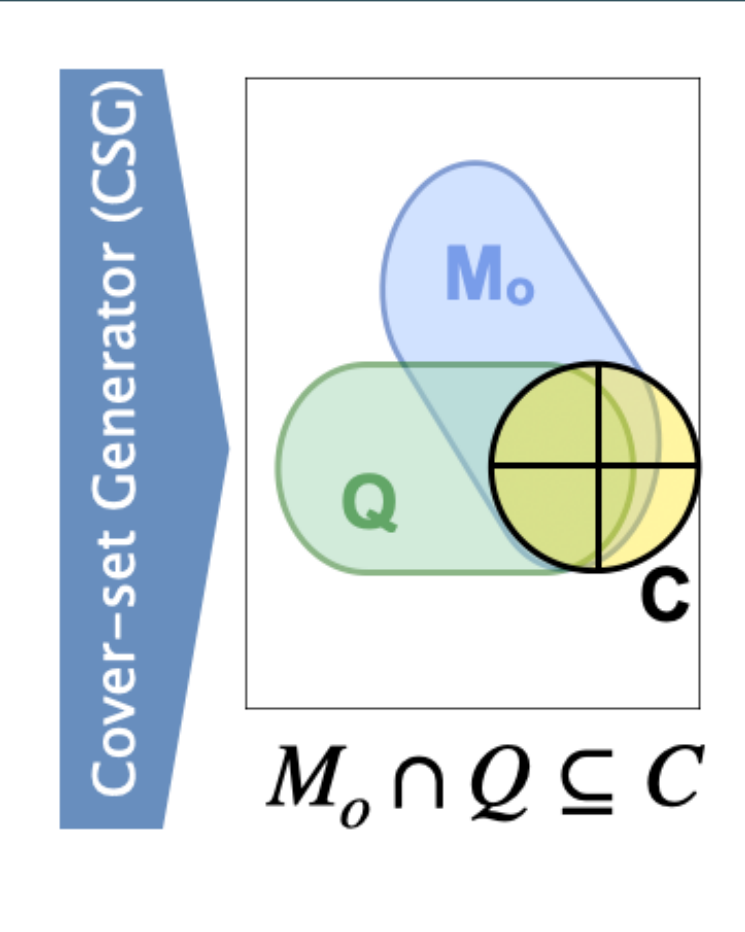
Over-approximate QuASI step 2: MUC generates a representative abstract trace for each cover-set component



Over-approximate QuASI step 2: If the representative trace is not consistent with input counts, then no trace in the cover-set component is

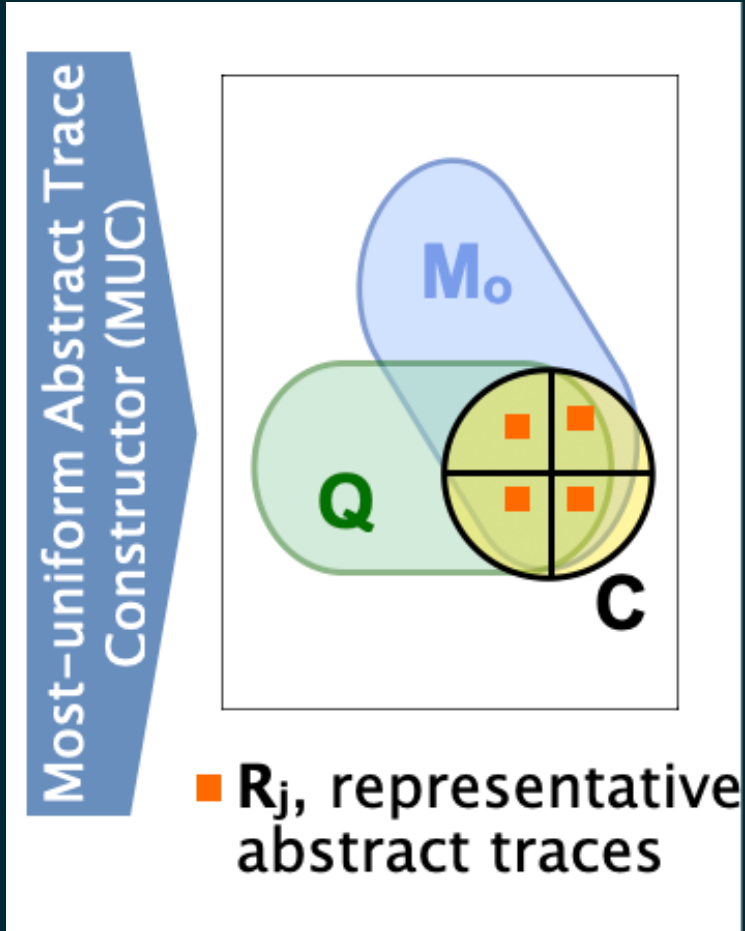


Over-approximate QuASI step 2: The representative abstract trace is the most uniform allocation subject to the constraints in the component

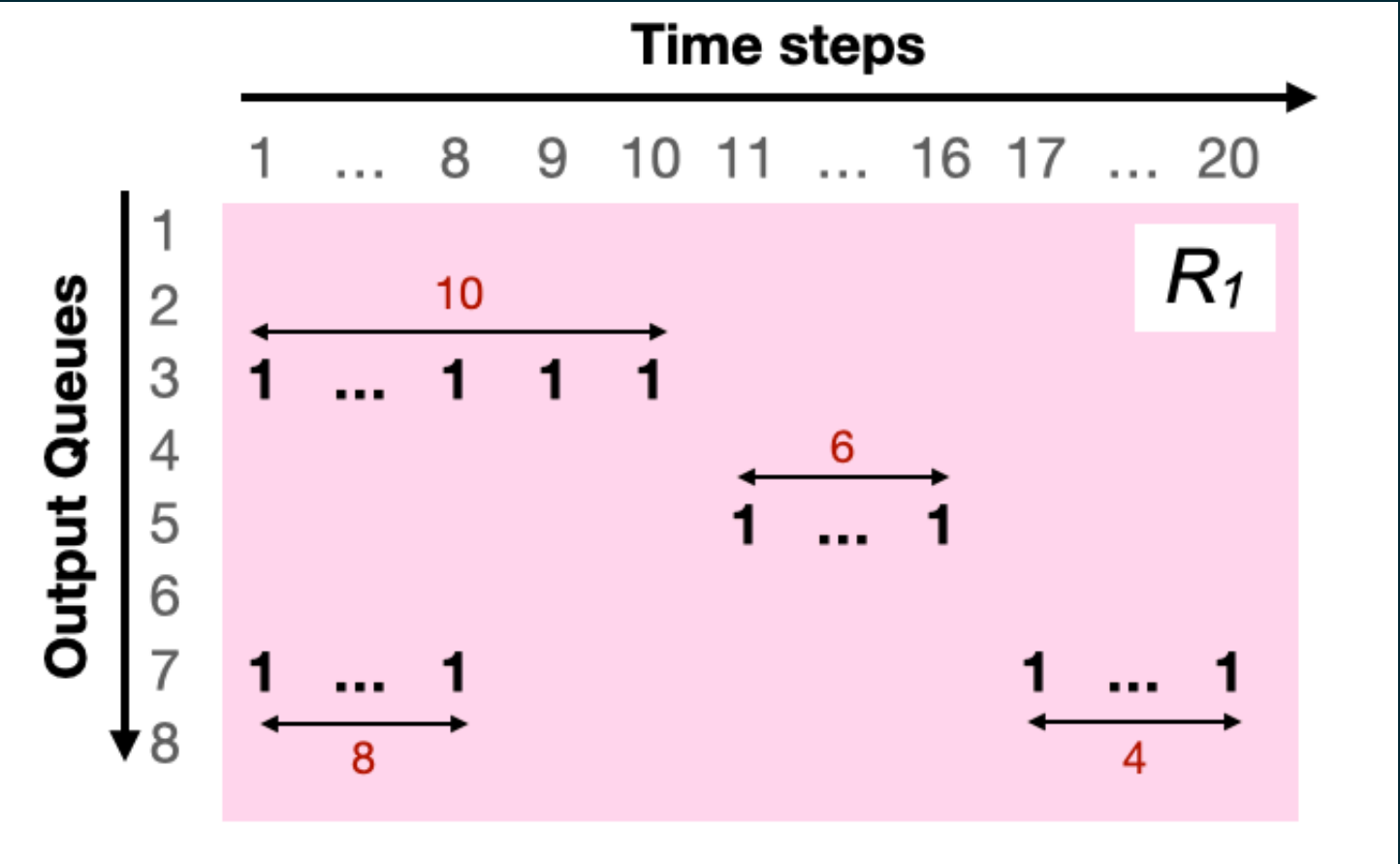


cover-set component F_C^1

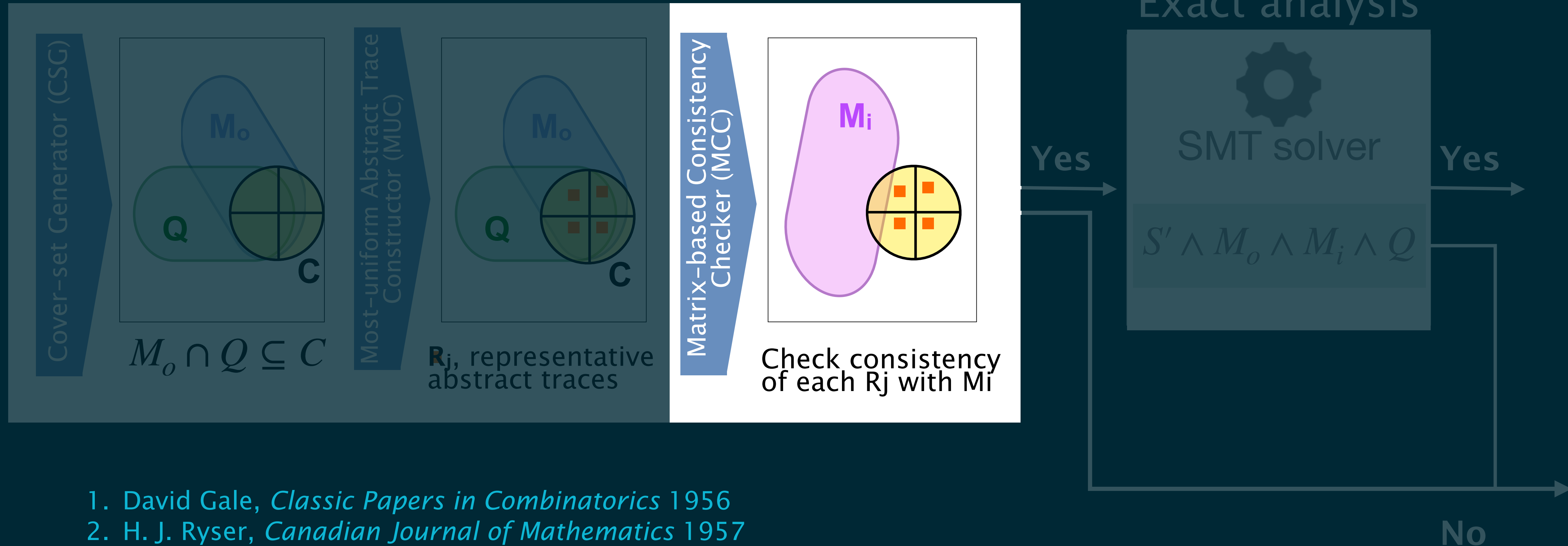
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representative abstract trace R1

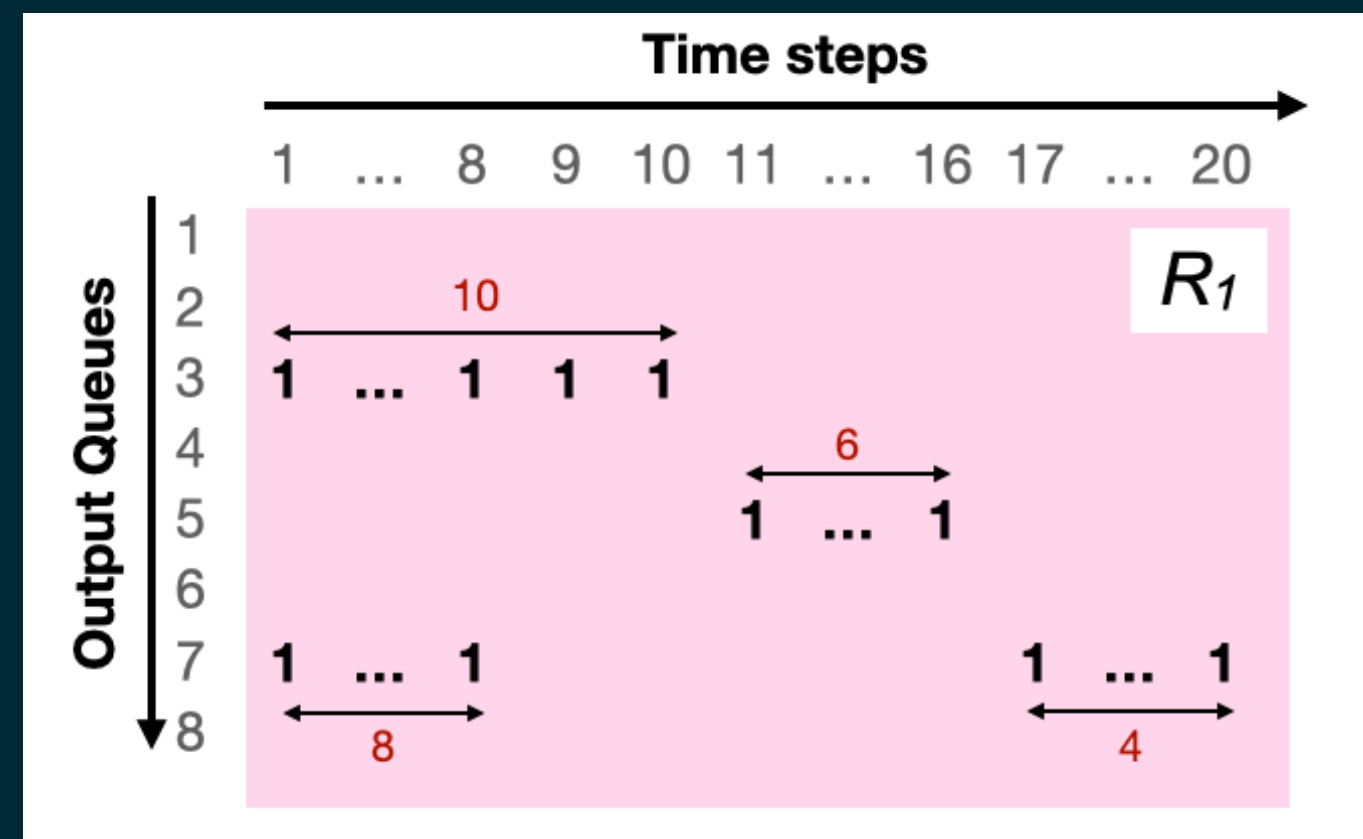


Over-approximate QuASI step 3: QuASI checks consistency of each representative abstract trace with M_i using the Gale-Ryser theorem^{1,2}



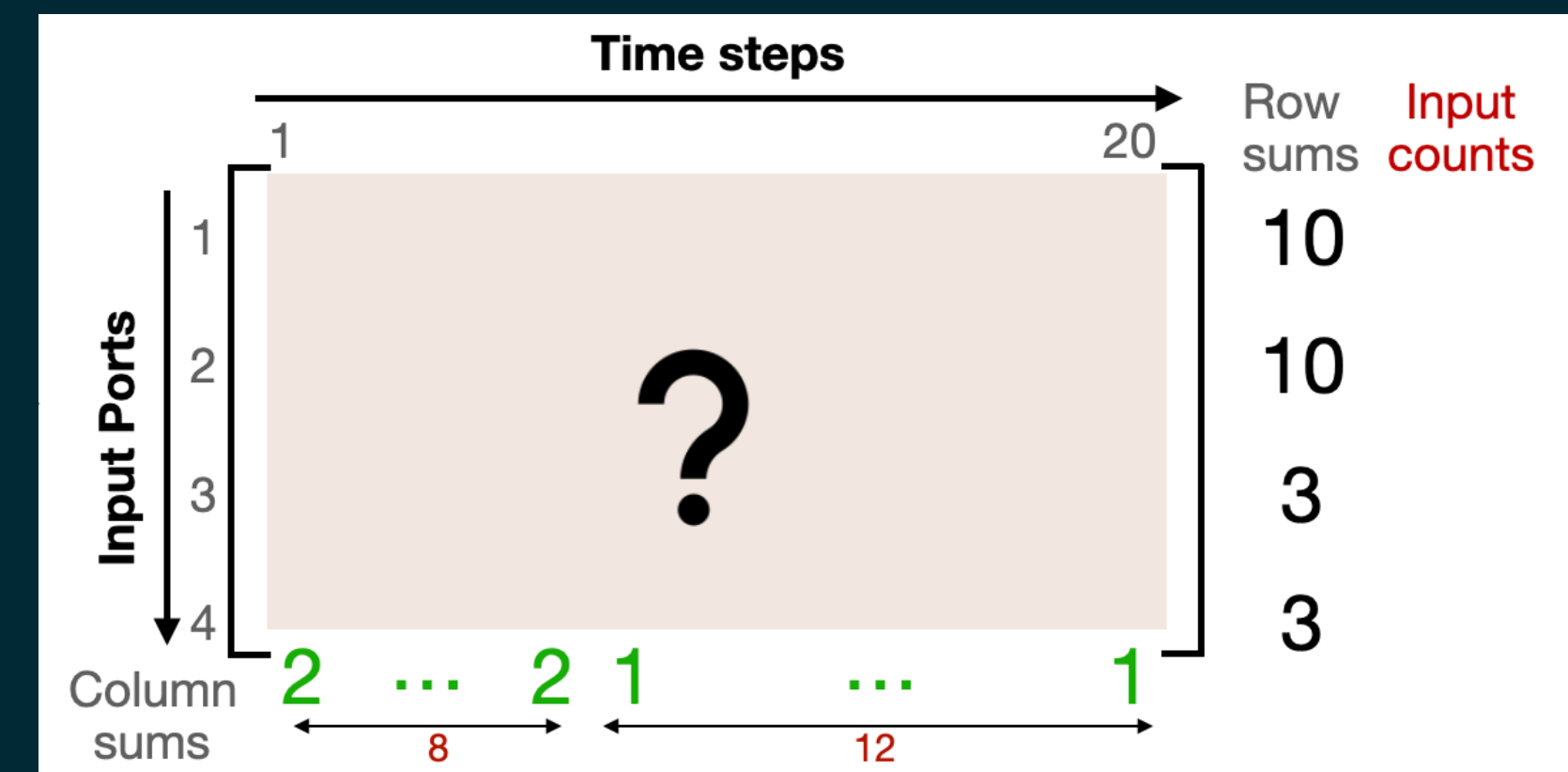
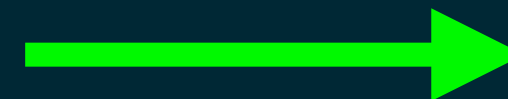
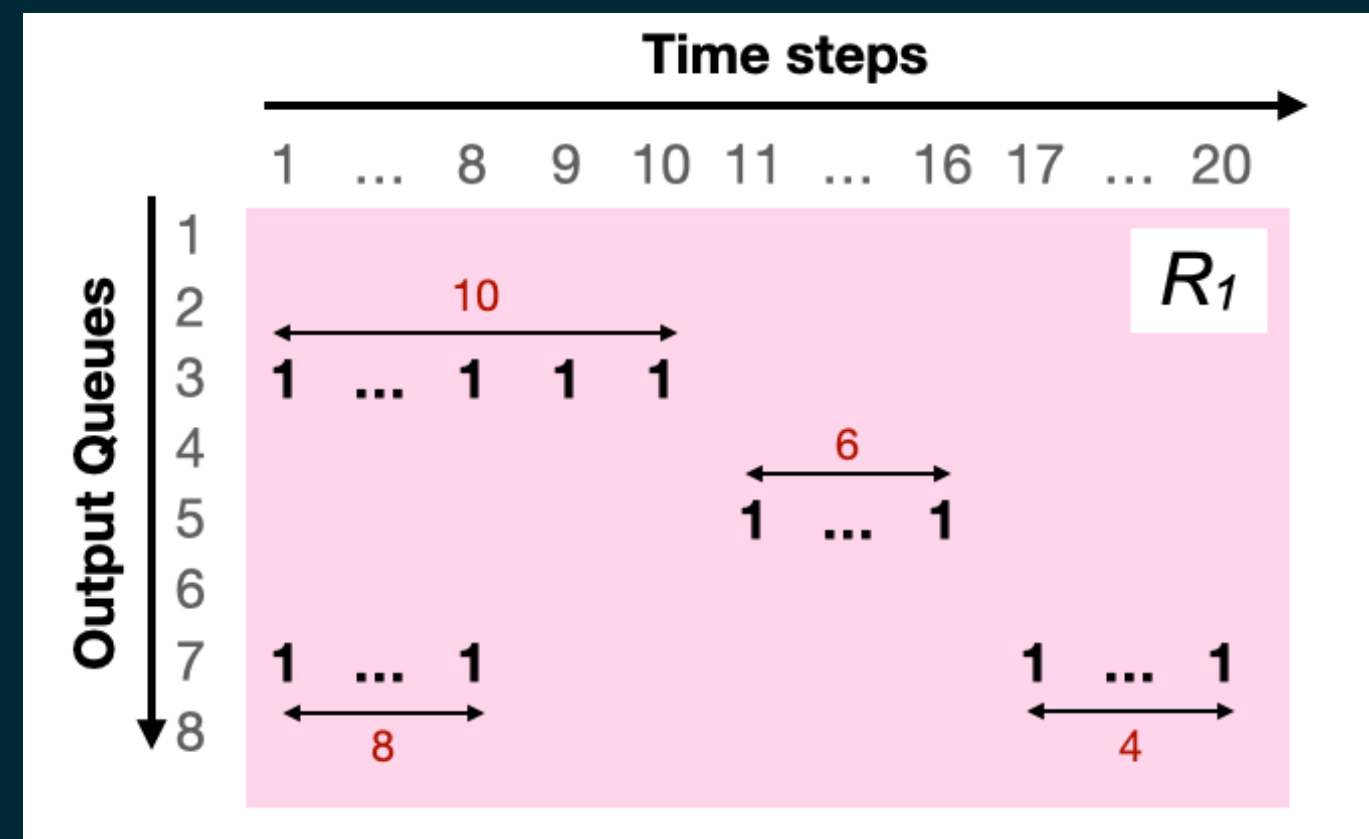
1. David Gale, *Classic Papers in Combinatorics* 1956
2. H. J. Ryser, *Canadian Journal of Mathematics* 1957

Over-approximate QuASI step 3: QuASI checks consistency of each representative abstract trace with M_i using the Gale-Ryser theorem^{1,2}



Is the R_1 representative trace consistent with the input counts?

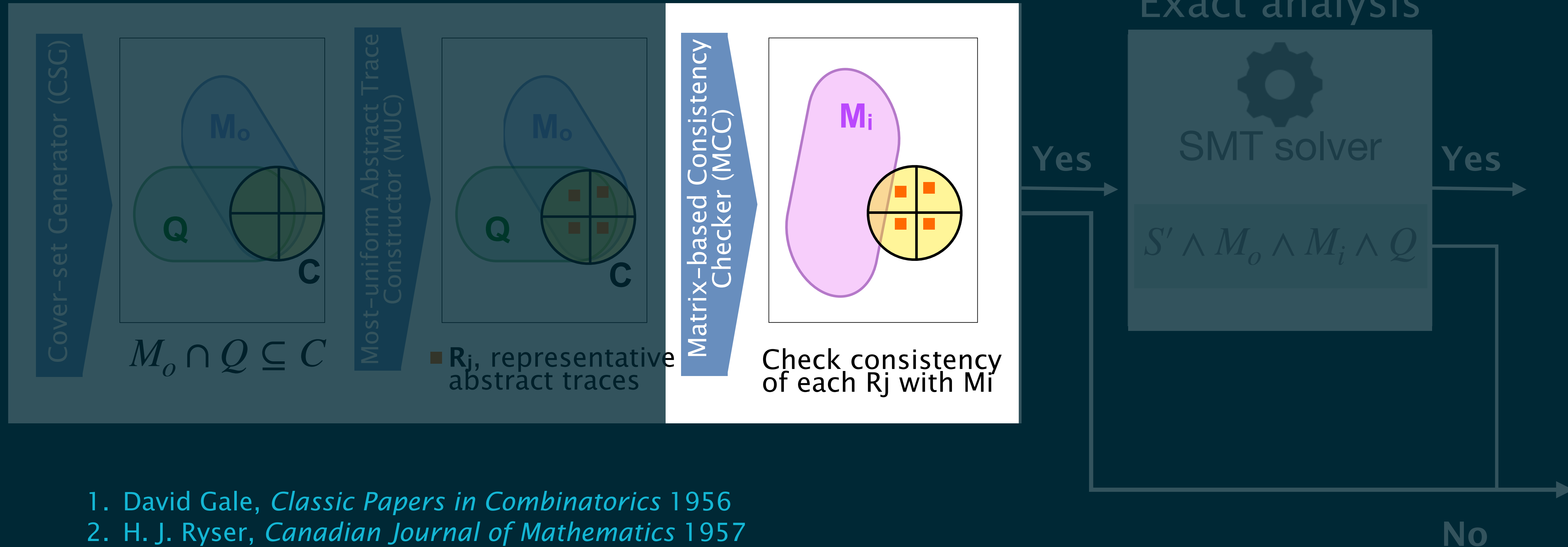
Over-approximate QuASI step 3: QuASI checks consistency of each representative abstract trace with M_i using the Gale-Ryser theorem^{1,2}



Is the R_1 representative trace consistent with the input counts?

Is there a (0,1)-matrix with row sums eq. input packet counts and column sums eq. total number of packets sent /time steps?

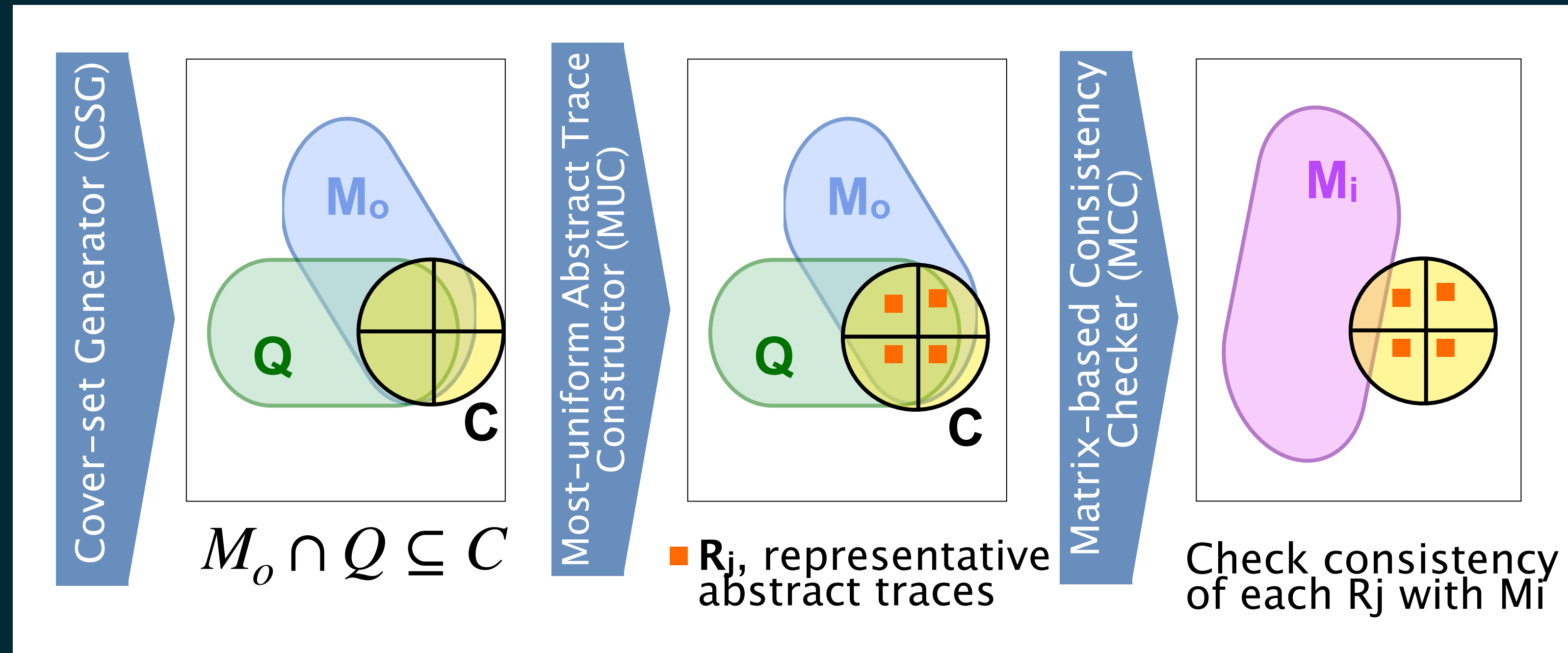
Over-approximate QuASI step 3: QuASI checks consistency of each representative abstract trace with M_i using the Gale-Ryser theorem^{1,2}



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If Gale-Ryser theorem says none of the representative traces is consistent with the input counts M_i , QuASI concludes— we are done!

Over-approximate analysis



Exact analysis

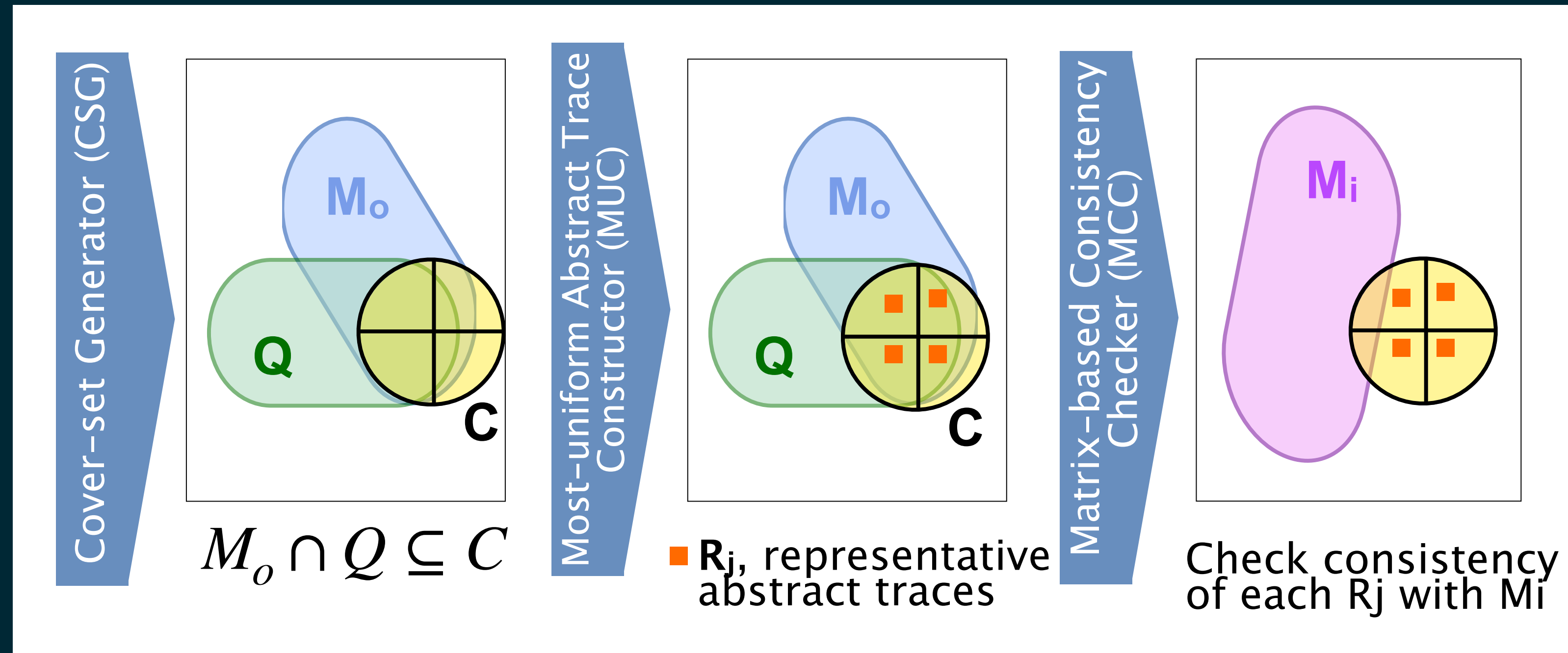


SMT solver

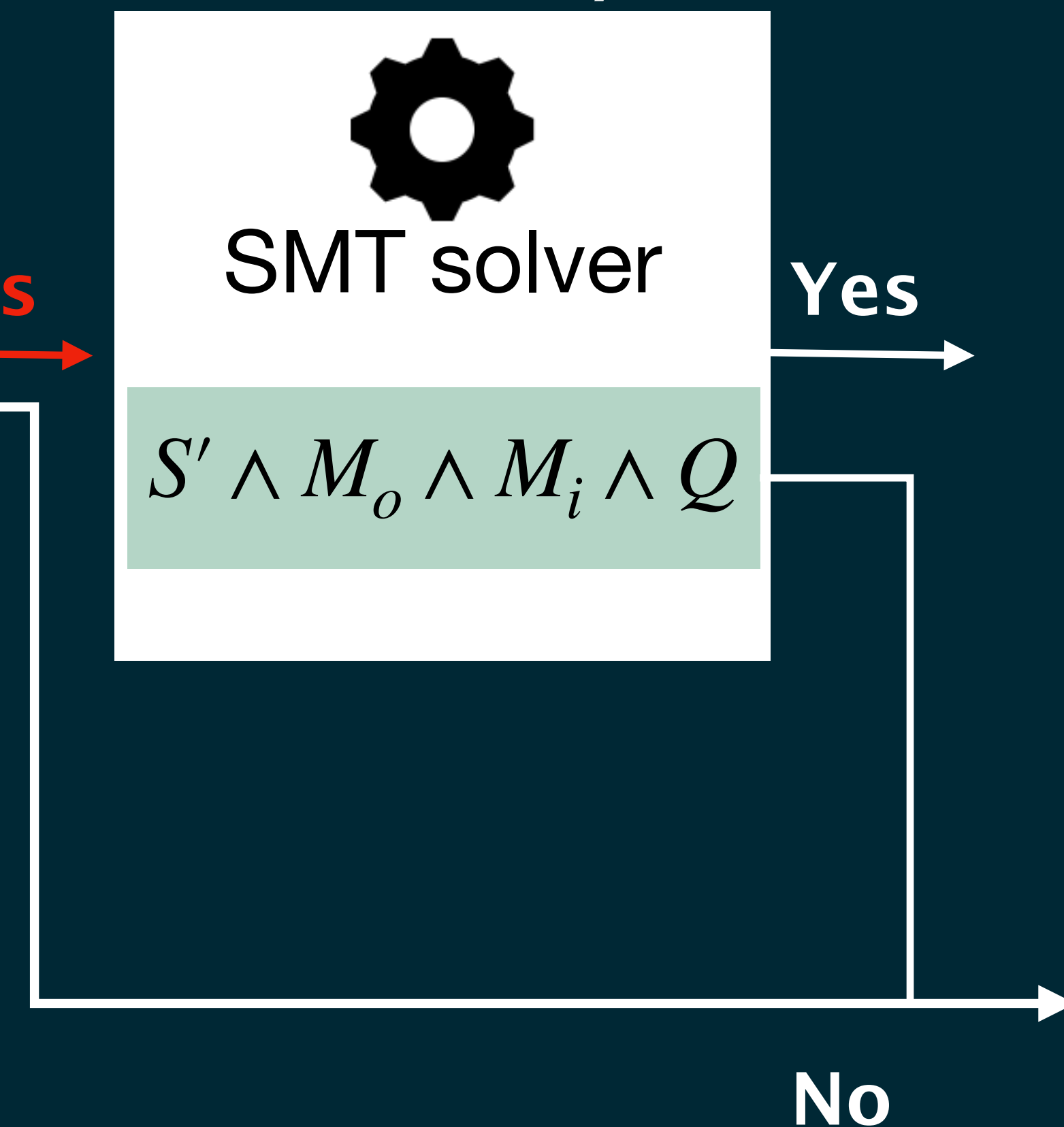
$$S' \wedge M_o \wedge M_i \wedge Q$$

Else, QuASI uses an SMT solver to model the exact conditions for consistency with M_i , M_o , and Q

Over-approximate analysis



Exact analysis



We evaluate QuASI in multiple queue-related queries

BurstOccurrence: Could a burst of rate R and duration D occur?

QlenK: Could the queue length at port O_i be at least K ?

MaxQlen: What is the maximum queue length at port I ?

MaxBuff: What is the maximum buffer occupancy during the interval?

QuASI evaluation

Is QuASI useful?

Is QuASI better
than SOTA?

Is QuASI scalable?

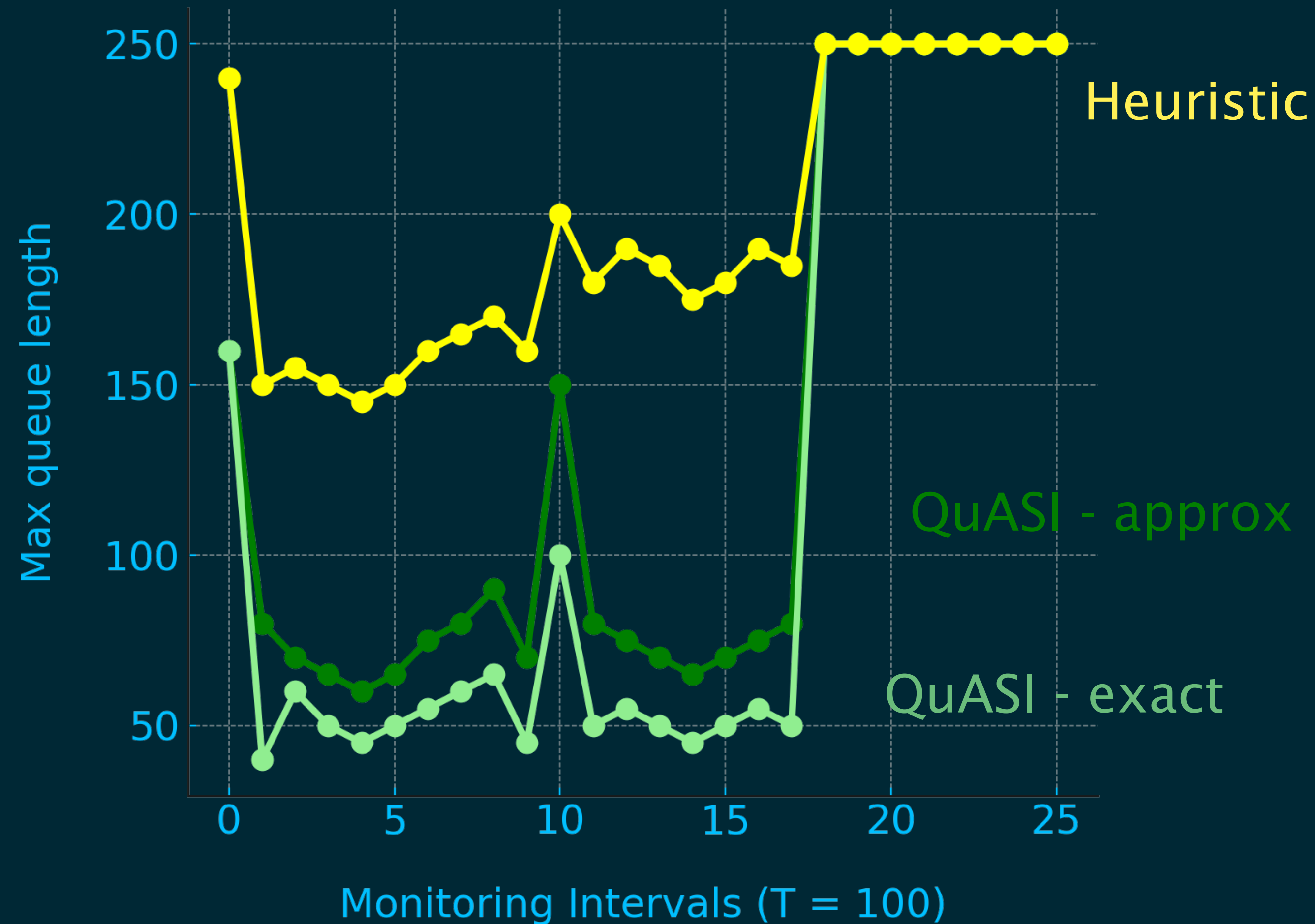
QuASI evaluation

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Is QuASI scalable?

QuASI-approx finds bounds up to **58% tighter** than the heuristic baseline within **1 second**



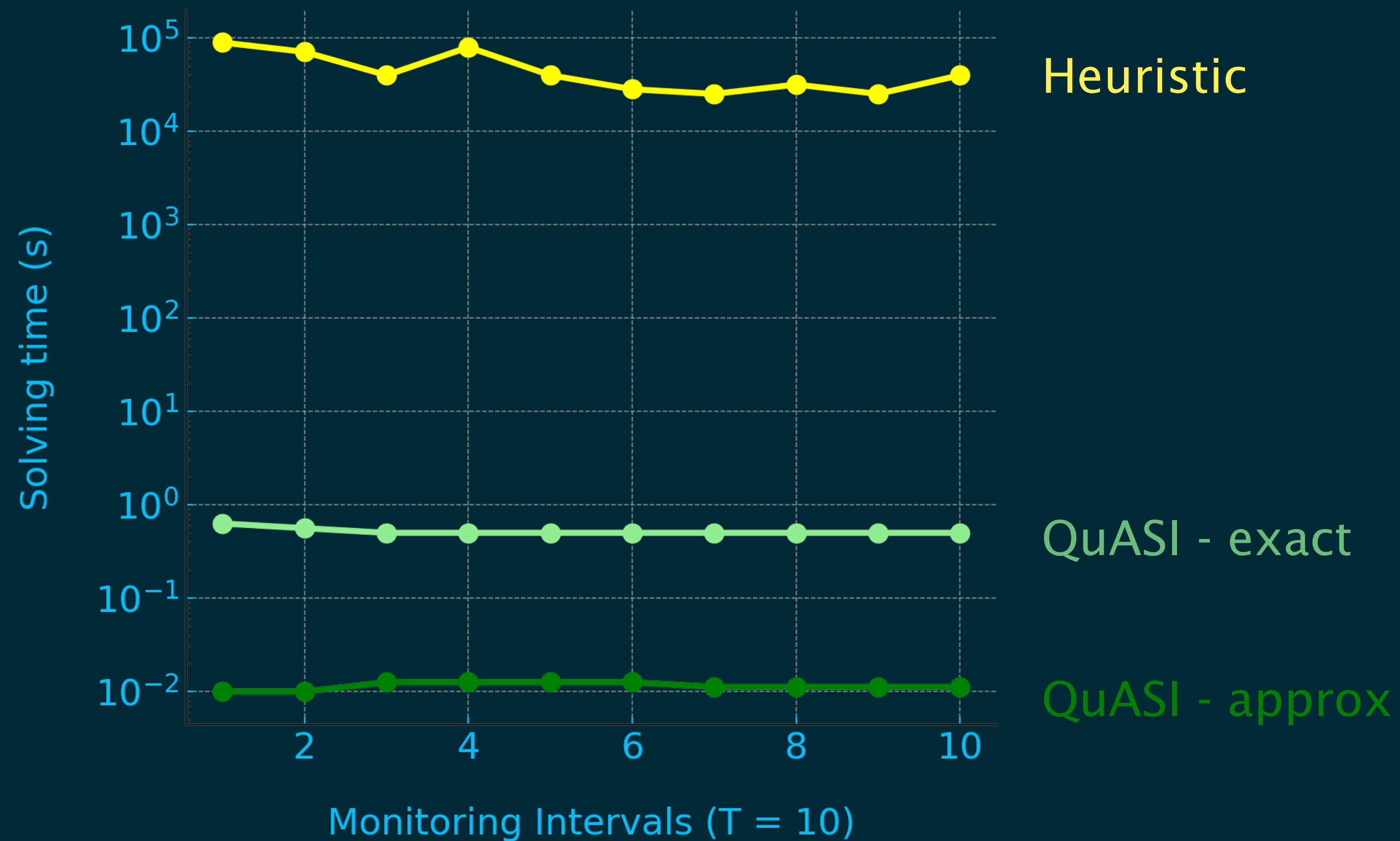
QuASI evaluation

Is QuASI useful?

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Is QuASI scalable?

QuASI computes the maximum queue length $10^6\times$ faster than FPerf [1]



[1] Arashloo et al. NSDI 2023

QuASI evaluation

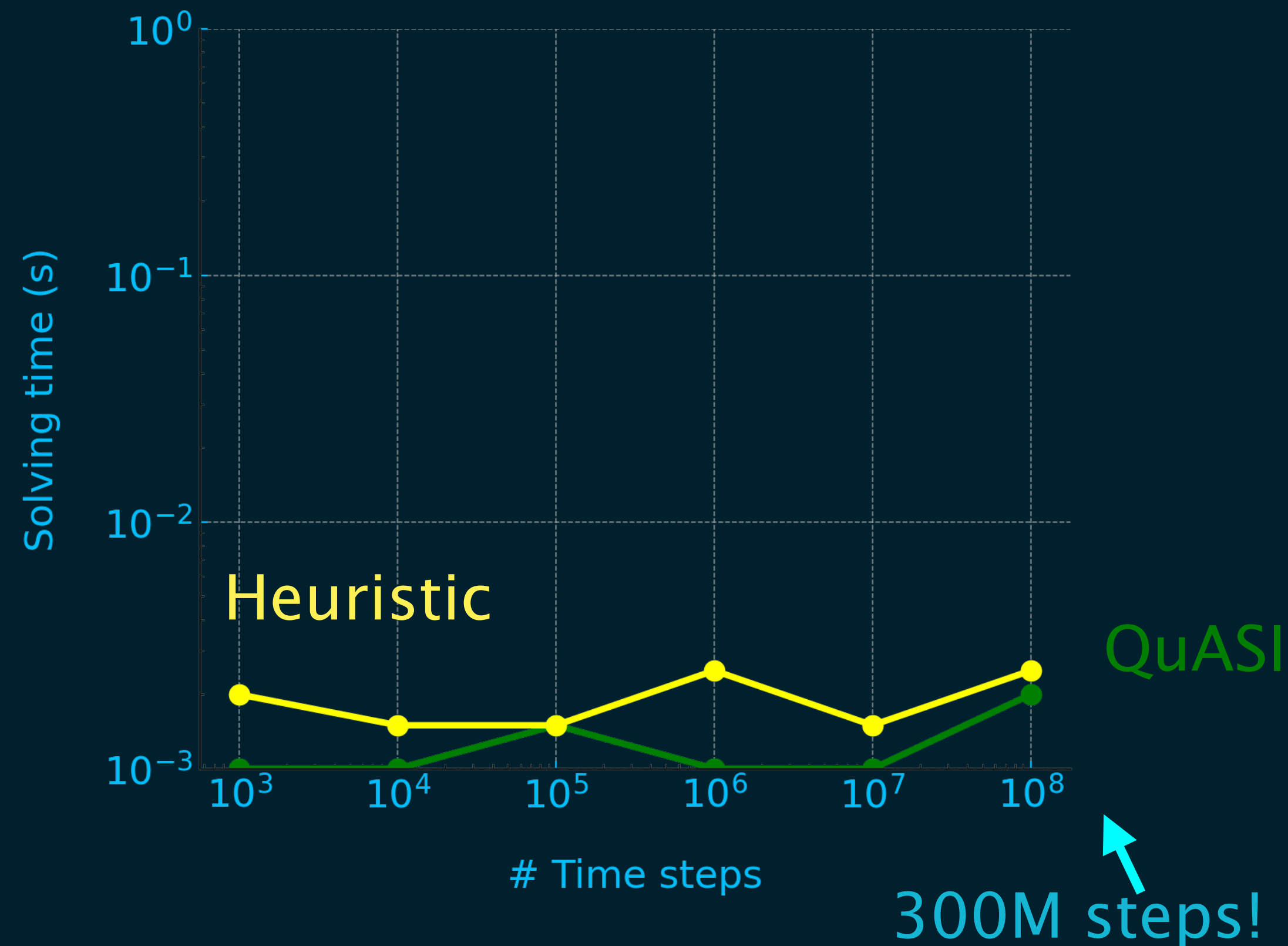
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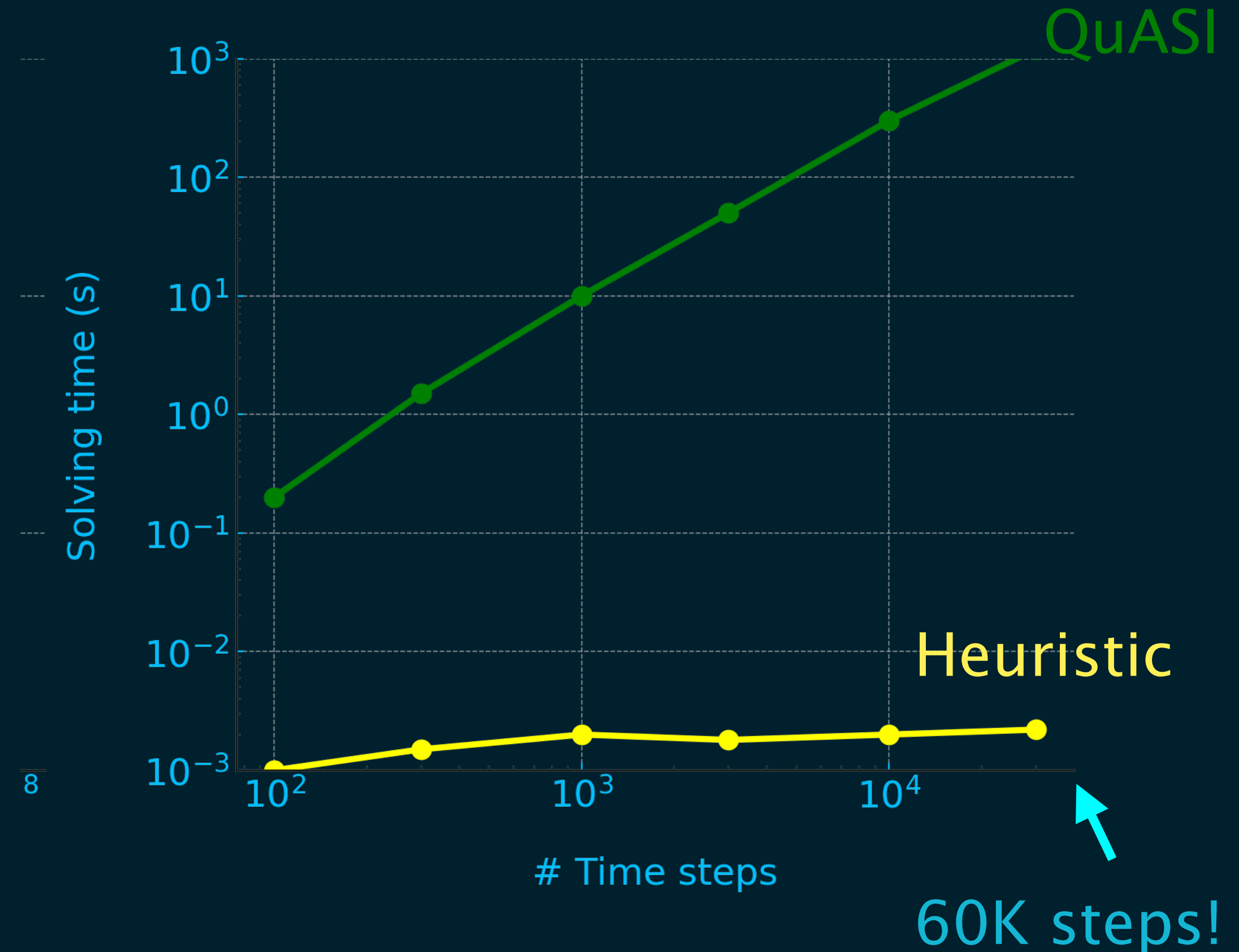
Is QuASI scalable?

QUASI takes almost constant time for QlenK
and scales quadratically with interval size for BurstOccurrence.

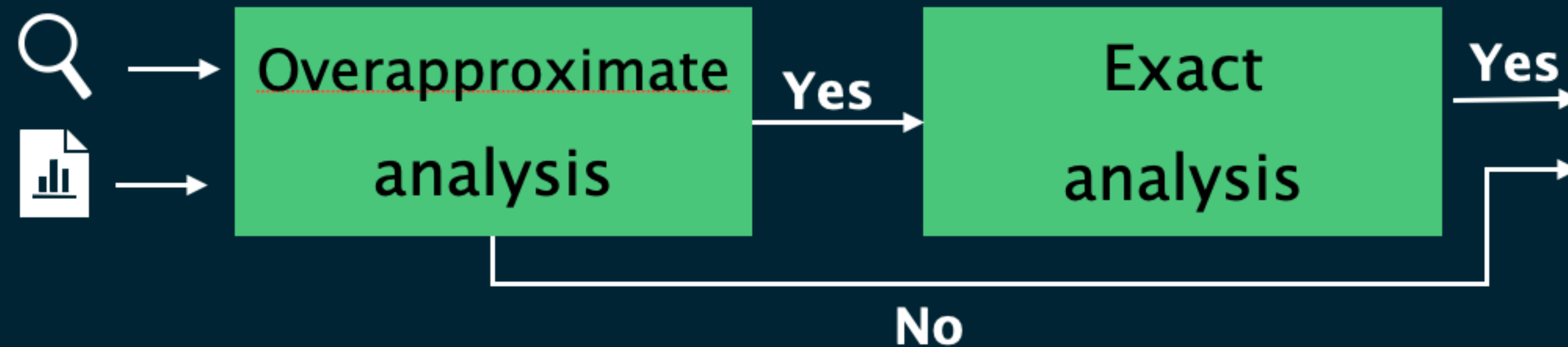
QlenK



Burst Occurrence

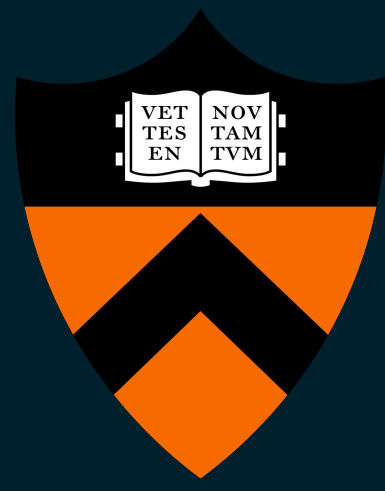


A Layered Formal Methods Approach to Answering Queue-related Queries



- ❖ QuASI answers queue-related (e.g., Burst occurrence) queries using coarse-grained per-port packet counts
- ❖ QuASI uses the enqueue rate abstraction which is lossless for the queries it supports
- ❖ QuASI is six orders of magnitude faster than SOTA while giving non-trivial answers





A Layered Formal Methods Approach to Answering Queue-related Queries

Divya Raghunathan, Maria Apostolaki, Aarti Gupta